Constrained min/max via Lagrange multipliers.

1. Let $C$ be the curve in $\mathbb{R}^2$ given by $x^3 + y^3 = 16$.
   (a) Sketch the curve $C$.
   (b) Is $C$ bounded?
   (c) Is $C$ closed?

2. Consider the function $f(x, y) = e^{xy}$ on $C$.
   (a) Is $f$ continuous? What does the Extreme Value Theorem tell you about the existence of global min and max of $f$ on $C$?
   (b) Use Lagrange multipliers to determine both the min and max values of $f$ on $C$.

3. Consider the surface $S$ given by $z^2 = x^2 + y^2$
   (a) Sketch $S$.
   (b) Use Lagrange multipliers to find the points on $S$ that are closest to $(4, 2, 0)$.

4. For the function shown on the back of the sheet, use the level curves to find the locations and types (min/max/saddle) for all the critical points of the function:

   \[ f(x, y) = 3x - x^3 - 2y^2 + y^4 \]

   Use the formula for $f$ and the 2nd-derivative test to check your answer.

5. If the length of the diagonal of a rectangular box must be $L$, what is the largest possible volume?