1. Here’s an example where one doesn’t have a local-to-global principle. Consider the equation \( x^2 + 17y^2 = 257 \). Show:
   
   (a) The equation has a solution in \( \mathbb{F}_p \) for all \( p \).
   
   (b) The equation has a solution in \( \mathbb{Z}_p \) for all \( p \).
   
   (c) Sadly, the equation has no solution in \( \mathbb{Z} \).

   For (a) and (b), the harder cases are when \( p \in \{2, 17, 257\} \). For part (b) with \( p = 2 \), one approach is to use Theorem 7.32 from page 114 of Milne’s notes.

2. Suppose \((V, B)\) is a quadratic space over a field \( K \) with \( \text{char}(K) \neq 2 \), and that \( B \) is nondegenerate. Let \( W \) be a subspace of \( V \). Show that:
   
   (a) \( (W^\perp)^\perp = W \)
   
   (b) \( \dim W + \dim W^\perp = \dim V \).
   
   (c) Show that if \((W, B)\) is nondegenerate, then \( V = W \oplus W^\perp \), where \( \oplus \) denotes orthogonal direct sum.
   
   (d) Give an example where (c) fails if \((W, B)\) is degenerate. Here \((V, B)\) should still be nondegenerate.

3. Let \((V, B)\) be a 2-dimensional quadratic space over \( K \). Prove that \( V \) is isotropic if and only if \( -\text{disc}(B) \) is a square in \( K \).

4. Suppose a real symmetric \( 4 \times 4 \) matrix \( G \) has characteristic polynomial \( x^4 - dx^2 + 12 \) for some \( d \in \mathbb{R} \). Let \( B \) be the corresponding bilinear form on \( \mathbb{R} \). Is \( B \) non-degenerate? Is it isotropic or anisotropic? What is its canonical form among those listed in class? (These things may depend on \( d \), and note that not all \( d \) are possible.)

5. Let \((V, B)\) be a nondegenerate quadratic space. Let \( x \) and \( y \) be anisotropic vectors in \( V \) with \( q(x) = q(y) \). Show that there is an isometry \( \tau \) of \( V \) such that \( \tau(x) = y \).