Math 530: Problem Set 6

Due date: In class on Friday, March 20.
Course Web Page: http://dunfield.info/530
Warning: This assignment is longer than it appears; the first problem has not less than 9 parts.

1. Marcus, Chapter 4, Problem 17.
2. Marcus, Chapter 4, Problem 27.
3. Let $K$ be a number field, $t > 0$ in $\mathbb{R}$. Show that the convex, centrally symmetric set

$$X = \left\{ (z_\tau) \in K^r \mid \sum_{\tau} |z_\tau| < t \right\}$$

has (canonical) volume $2^r \pi^s t^n / n!$. Hint: One approach is to induct on $r$ and $s$.

4. Let $a$ be an ideal of $\mathcal{O}_K$. Using only theorems stated in class, prove there exists an $a \neq 0$ in $\mathcal{O}_K$ such that

$$| \mathcal{N}_{K/Q}(a) | \leq M \mathcal{N}(a)$$

where $M = \frac{n!}{\pi^n} \left( \frac{4}{n} \right)^s \sqrt{|\Delta_K|}$.

This is the Minkowski Bound, infamous on the 530 comp exams.

Hint: Use problem 3 and the inequality between the arithmetic and geometric means:

$$\frac{1}{n} \sum_{\tau} |z_\tau| \geq \left( \prod_{\tau} |z_\tau| \right)^{1/n}$$