Math 525: Problem Set 2

Due date: In class on Wednesday, September 9.
Course Web Page: http://dunfield.info/525
Office hours: Mondays from 11-12, Tuesdays from 11:15 - 12:15, and by appointment. For an appointment, just talk to me after class, or email me at nmd@illinois.edu.
Required Text: Allen Hatcher, *Algebraic Topology*,
http://www.math.cornell.edu/~hatcher/AT/ATpage.html

1. Recall that a topological space $X$ is connected if it is not the disjoint union of two non-empty open sets. The space $X$ is path-connected if every pair of points can be joined by a path.

   (a) Prove directly from the least upper bound property that $\mathbb{R}$ is connected.

   (b) Is every path-connected space is also connected? Prove your answer. (In class, an example was give of a connected space that’s not path-connected)

2. A map $p: \tilde{X} \to X$ is a local homeomorphism if for every $\tilde{x} \in \tilde{X}$ has an open neighborhood $U$ so that $p|_U$ is a homeomorphism.

   (a) Prove that if $\tilde{X}$ is compact and Hausdorff$^2$, then $p$ is a covering map. (While every covering map is a local homeomorphism, the converse isn’t always true as we saw in class.)

   (b) Again assuming $\tilde{X}$ is compact and Hausdorff, prove that for each $x_0 \in X$, the set $p^{-1}(x_0)$ is finite. If $X$ is path connected, show that the number of points in $p^{-1}(x_0)$ is independent of the choice of $x_0$. The size of $p^{-1}(x_0)$ is called the degree of $p$.

3. Hatcher, Section 1.3, Problem 1.


N.B. The problems removed from this assignment will appear on the next problem set.

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$^1$Revised version of September 8.

$^2$A topological space is Hausdorff if for every two points $x$ and $y$ there are disjoint open sets $U$ and $V$ with $x \in U$ and $y \in V$. A metric space is always Hausdorff, and we will rarely consider spaces which don’t have this property.