Lecture 29: Midterm review — exam tomorrow.

Note: curl not on the exam.

---

Parameterizations: Ch. 3 Review Exercise #12

Let's start with param the big circle $c_1 : [0, T] \to \mathbb{R}^2$

Start with some param $c_0 : [0, 2\pi] \to \mathbb{R}^2$ of big circle $c_0(t) = (R \cos t, R \sin t)$

$h(0) = 0$
$h(T) = 2\pi$
$h' > 0$

$c_1(t) = (R \cos (\frac{2\pi}{T}) t, R \sin (\frac{2\pi}{T}) t)$
\[ C_2 : [0, T] \rightarrow \mathbb{R}^2 \]
\[ C_2(t) = (r \cos \left( \frac{8\pi}{T} t \right), r \sin \left( \frac{3\pi}{T} t \right)) \]

\[ C : [0, T] \rightarrow \mathbb{R}^2 \]
\[ C(t) = C_1(t) + C_2(t) = (R \cos \frac{2\pi t}{T} + r \cos \frac{8\pi t}{T}, R \sin \frac{2\pi t}{T} + r \sin \frac{3\pi t}{T}) \]

Take \( R = 2 \) \( r = 1 \) \( T = 2\pi \)
\[ C(t) = (2 \cos t + \cos 4t, 2 \sin t + \sin 4t) \]
\[ C'(t) = (-2 \sin t - 4 \sin 4t, 2 \cos t + 4 \cos 4t) \]

\[ ||C'(t)|| = \sqrt{20 + 16 \cos 3t} \]
min speed: \( 2 \)
max speed: \( 6 \)

Length = \[ \int_0^{2\pi} ||C'(t)|| \, dt \approx 26.72 \]

\[ f(x, y) = x^2 \]
\[ \int_C f \, ds = \int_0^{2\pi} f(C(t)) ||C'(t)|| \, dt \]
\[ = \int_0^{2\pi} (2 \cos t + \cos 4t)^2 \sqrt{20 + 16 \cos 3t} \, dt \approx 78.98 \]

\[ F = (y, x) \]
\[ \int_C F \cdot ds = \int_0^{2\pi} F(C(t)) \cdot C'(t) \, dt = \int_0^{2\pi} \nabla \phi \cdot \nabla \phi \, dt \]
\[ = 0 \quad \text{so} \quad \Phi \text{ is conservative!} \]
Changing speeds

Suppose \( c \) is unit speed, e.g. \( c(t) = (\cos t, \sin t) \).

In this case the \( t \) parameter in \( c(t) \) can equally be viewed as
time or distance from the starting point.

\( h \) is the instructions of the form \( h(t) = \text{milepost} \).

Suppose we want a param of the curve
with non-constant speed, need something with

\[
\begin{align*}
h(0) &= 0 \\
h(1) &= 1 \\
h(2) &= 3 \\
h(\pi) &= 2\pi
\end{align*}
\]

or really anything

that is not a
straight line.
Lagrange multi:

6 m\(^2\) of material
max volume.

\[ V = xyz \]

\[ A = 2xy + 2xz + 2yz = 6 \quad x, y, z > 0 \]

\[ \nabla V = \lambda \nabla A \]

\[ (yz, zx, xy) = x^2(y+z, x+z, x+y) \]

\[ \Rightarrow \frac{yz}{y+z} = \frac{xz}{x+z} \Rightarrow (x+z)(y^2z = (y+z)x^2 \]

\[ yz^2 = xz^2 \Rightarrow x = y \]

Also

\[ \frac{yz}{y+z} = \frac{xy}{x+y} \Rightarrow y^2z = xy^2 \Rightarrow x = z \]

So \[ x = y = z \] and \[ A = 6 \Rightarrow 6x^2 = 6 \Rightarrow x = 1 \]
\[ y = 1 \]
\[ z = 1 \]

So only one int pt.

Still need to deal with when \[ x, y, z \] are small to show there is a global max.
Plane given by $\mathbf{r} \cdot \mathbf{n} = d$

Minimize $f(x,y,z) = x^2 + y^2 + z^2$ on $\mathbf{n}$ given that such a minimum exists.

$\nabla f = \lambda \nabla g = \lambda (a, b, c)$

$(2x, 2y, 2z) \Rightarrow 2x = \lambda a$  
$2y = \lambda b$  
$2z = \lambda c$

$g = 0 \Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) + d = 0$

$\Rightarrow x \frac{x}{2} (a^2 + b^2 + c^2) = -d \Rightarrow \frac{a^2 b^2 c^2}{2}$

$\lambda = \frac{-2d}{a^2 + b^2 + c^2}$

$x = \frac{-ad}{a^2 + b^2 + c^2}$,  
$y = \frac{-bd}{a^2 + b^2 + c^2}$

$z = \frac{-cd}{a^2 + b^2 + c^2}$. 