Math 241 F1H: Problem Set 9

Due date: Tuesday, April 8.

Midterm: The third midterm exam will be held in class on Thursday, April 10.

Office Hours: My office hours next week will be
- Monday: 3-5.
- Tuesday: 9–10:30 and 4:00–5:30.
- Wednesday: 9–10:30 and 3:00–5:00.

Review: Wednesday’s lecture will be a review session. As always, please send me suggestions for topics.

1. Section 7.3: #16.

2. Section 7.3: #17.

3. Section 7.4: #3, 5, 6.

4. Section 7.4: #15. (See Example 2.90 on page 146 for the setup on heat flow. In this problem, the conductivity is to be taken to be 1.)

5. Section 7.4: #21.

6. Section 7.4: #23.

7. Verify Green’s Theorem for the region and vector field given in §8.1 #12, that is, compute the line integral \( \int_C \mathbf{F} \cdot ds \) directly and compare the result to \( \iint_D \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \, dA \).

8. Let \( D \) be a region in \( \mathbb{R}^2 \) which is star-shaped about \( 0 \). That is, the boundary of \( D \) can be parameterized by

\[
c(t) = \left( f(t) \cos t, f(t) \sin t \right) \quad \text{for} \quad 0 \leq t \leq 2\pi,
\]

where \( f: [0, 2\pi) \to [0, \infty) \) satisfies \( f(0) = f(2\pi) \) so that this gives us a closed curve.

Without using Green’s Theorem, show that for the vector field \( \mathbf{F} = \frac{1}{2}(-y, x) \) one has

\[
\int_{\partial D} \mathbf{F} \cdot ds = \text{Area}(D)
\]

by using the parameterization above.

9. Section 8.2: #2.

10. Section 8.2: #7.


12. Consider a region \( D \) in \( \mathbb{R}^2 \) with a single boundary component \( C \), and let \( \mathbf{n} \) be the outward-pointing unit vector field.

Given a vector field \( \mathbf{F} = (F_1, F_2) \) on \( D \), define a new vector field by \( \mathbf{G} = (-F_2, F_1) \). Show that
(a) \( \int_C \mathbf{F} \cdot \mathbf{n} \, ds = \int_C \mathbf{G} \cdot ds \)

(b) \( \iint_D \text{div} \mathbf{F} \, dA = \iint_D \left( \frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y} \right) \, dA. \)

Explain why this means that Green's Theorem and the Divergence Theorem in \( \mathbb{R}^2 \) are equivalent.

Note: This assignment is complete.