Math 231 E1H: HW #12

**Due date:** In class on Wednesday, December 10.

Smith and Minton, Section 9.6: #13, 15, 17.
Hubbard and Hubbard (attached) Section 0.7: #2, 3, 6, 8, 11(a), 13.

Math 231 E1H: Honors Problem Set #4 (Corrected)

**Due date:** In class on Wednesday, December 10, on separate sheets from HW #12.

1. Recall that a function is *odd* if \( f(-x) = -f(x) \) for all \( x \), and *even* if \( f(-x) = f(x) \) for all \( x \).
   Suppose \( f \) and \( g \) are two functions and set \( h(x) = f(x)g(x) \).
   
   (a) Suppose \( f \) is odd and \( g \) is even. What can you say about \( h \)?
   (b) Suppose \( f \) and \( g \) are both odd. What can you say about \( h \)?
   (c) Suppose \( f \) and \( g \) are both even. What can you say about \( h \)?

2. Suppose \( f \) is an odd function. Show that for each \( L > 0 \) one has
   \[
   \int_{-L}^{L} f(x) \, dx = 0.
   \]
   Hint: break the integral into two pieces by splitting the interval \([-L, L]\) at 0. Then do a change of variables (\( u \)-substitution) to one of the new integrals to make it look more like the other one.

3. Suppose \( f \) is an even function. Find a relationship between
   \[
   \int_{-L}^{L} f(x) \, dx \quad \text{and} \quad \int_{0}^{L} f(x) \, dx.
   \]
   Justify your answer carefully.

4. Use questions 1 and 2 to prove that if \( f \) is an odd function then its Fourier expansion has no cosine terms (i.e. \( a_k = 0 \) for \( k > 1 \)). What, if anything, can you say about \( a_0 \)?

5. Use the properties that \( e^{a+b} = e^a e^b \) and \( e^{i\theta} = \cos \theta + i \sin \theta \) for a real number \( \theta \) to derive the sum formulas for sine and cosine.