Exercise 0.1. Find the order of 3 modulo 13.

Exercise 0.2. Solve the following.
   a) Let $m$ be a positive integer and let $a$ and $b$ be integers relatively prime to $m$ with $(\exp_m a, \exp_m b) = 1$. Then prove that $\exp_m ab = (\exp_m a)(\exp_m b)$.
   b) Show that the hypothesis $(\exp_m a, \exp_m b) = 1$ cannot be eliminated from (a). What can be said about $(\exp_m ab)$ if $(\exp_m a, \exp_m b) \neq 1$?

Exercise 0.3. Let $m$ be a positive integer and let $d | \phi(m)$ with $d > 0$. Prove or disprove that there exists $a \in \mathbb{Z}$ with $\exp_m a = d$.

Exercise 0.4. Let $m$ be a positive integer and let $a \in \mathbb{Z}$ with $(a, m) = 1$.
   a) Prove that if $a \in \mathbb{Z}$ with $\exp_m a = xy$, with $x$ and $y$ positive integers, then $\exp_m a^x = y$.
   b) Prove that if $\exp_m a = m - 1$, then $m$ is a prime.

Exercise 0.5. Determine the number of incongruent primitive roots modulo 43.

Exercise 0.6. Let $p$ be an odd prime number.
   a) Prove that any primitive root $r$ modulo $p$ is a quadratic non-residue modulo $p$. Deduce that
      $$r^{(p-1)/2} \equiv -1 \mod p.$$  
   b) Prove that there are exactly $\frac{p-1}{2} - \phi(p-1)$ incongruent quadratic non-residues modulo $p$ that are not primitive roots modulo $p$.

Exercise 0.7. Let $p$ be a prime number. Prove that the product of all incongruent primitive roots modulo $p$ is congruent to $(-1)^{\phi(p-1)}$ modulo $p$. Deduce that if $p > 3$, then the product of all incongruent primitive roots modulo $p$ is congruent to 1 modulo $p$. 