Exercise 0.1. Determine (with a proof or a counterexample) whether each of the arithmetic functions below is completely multiplicative, multiplicative or both. Here $k$ is a fixed real number.

a) $f(n) = kn$,

b) $f(n) = n^k$.

Exercise 0.2. Let $n \in \mathbb{N}$. The Liouville $\lambda$-function, denoted by $\lambda(n)$, is

(0.1) $\lambda(n) = \begin{cases} 1, & \text{if } n = 1, \\ (-1)^k, & \text{if } n = p_1p_2\cdots p_k \text{ where } p_1, p_2, \cdots, p_k \text{ are not necessarily distinct prime numbers}. \end{cases}$

a) Prove that $\lambda$ is a completely multiplicative function.

b) Let $F(n)$ be

$$F(n) = \sum_{d|n} \lambda(d).$$

Prove that $F(n) = 1$ if $n$ is a perfect square and 0 otherwise.

Exercise 0.3. Solve the following.

a) Prove that there are infinitely many integers $n$ for which $\phi(n) = \frac{n}{3}$.

b) Prove that there are no integers $n$ for which $\phi(n) = \frac{n}{4}$.

Exercise 0.4. Prove that if $n$ is a positive integer, then

(0.2) $\phi(n) = \begin{cases} \phi(n), & \text{if } n \text{ is odd}, \\ 2\phi(n), & \text{if } n \text{ is even}. \end{cases}$

Exercise 0.5. Characterize those positive integers $n$ for which each of the following properties holds.

a) $d(n) = 1$,

b) $d(n) = 2$,

c) $d(n) = 3$.

Exercise 0.6. Characterize those positive integers $n$ for which $d(n)$ is odd.