5.2.8] Suppose a parallelogram \( P \) has a diagonal as a line of symmetry:

![Parallelogram Diagram]

so if \( P \) is the reflection across \( L \), then
\[
\begin{align*}
P(A) &= B, \\
P(D) &= D, \\
P(C) &= C.
\end{align*}
\]

Then \( AD \) is congruent to \( P(AD) = BD \)
and \( AC \) is congruent to \( P(AC) = BC \).

On the other hand, we proved on the midterm that opposite sides in a Euclidean parallelogram are congruent, so \( AC \cong BD \). Hence \( AD \cong BD \cong AC \cong BC \), as desired.

5.3.3] On the one hand, if \( T = r_1 o r_2 \) then
\[
(r_2 o r_1)(r_1 o r_2) = r_2 o (r_1 o r_1), r_2 = r_2 o d o r_2
= r_2 o r_2 = id, \text{ since for any reflection } r, r^{-1} = r.
\]

Similarly \( (r_1 o r_2) o (r_1 o r_1) = id \),
so \( T^{-1} = r_2 o r_1 \).

On the other hand, if
\[
T(\vec{w}) = \vec{w} + \vec{v} \text{ for every vector } \vec{w} \in \mathbb{R}^2, \text{ then }
\]
\[
T^{-1}(\vec{w}) = \vec{w} - \vec{v}.
\]

Indeed,
\[
(\vec{w} + \vec{v}) - \vec{v} = \vec{w}, \text{ so letting } S(\vec{w}) = \vec{w} - \vec{v},
\]
we get \( ST = id \equiv TO \cdot S \). So \( S = T^{-1} \).
5.3.5) If \( T_1 \) is defined by \( T_i(w) = w + v_i, \ i = 1, 2, \) then for every \( w, \)
\[
(T_2 \circ T_1)(w) = T_2(w + v_1) = (w + v_1) + v_2 = (w + v_2) + v_1
\]
\[
= T_1(w + v_2) = (T_1 \circ T_2)(w), \) so \( T_2 \circ T_1 = T_1 \circ T_2. \)

5.3.6) Suppose \( l \) has the form
\[
l = \{ \bar{p} + tv \mid t \in \mathbb{R} \} \) for vectors \( \bar{p}, \bar{v}, \)
and \( \bar{w} \) is a vector perpendicular to \( \bar{v}. \)
Suppose first that \( \bar{p} = 0. \) If \( \bar{p} \) is the reflection across \( l, \) then for any constants \( a \) and \( b, \)
\[
p(a\bar{v} + b\bar{w}) = a\bar{v} - b\bar{w}.
\]

If \( T \) is the translation \( T(x) = x + c\bar{v} \) for some constant \( c \in \mathbb{R}, \) then
\[
p \circ T(x) = p(x + c\bar{v}). \) Writing \( x = a\bar{v} + b\bar{w}, \)
\[
p(a\bar{v} + b\bar{w} + c\bar{v}) = p(a\bar{v} + (a+c)\bar{w}) = a\bar{v} + (a+c)\bar{w} - b\bar{w}
\]
\[
= a\bar{v} - b\bar{w} + c\bar{v} = p(a\bar{v} + b\bar{w}) + c\bar{v} = Tp(x).
\]
So \( p \circ T = Tp. \)

Now, suppose \( l \) does not pass through \( 0. \) Let
\[
T_{\bar{p}}(x) = x + \bar{p}. \) Then \( T_{-\bar{p}}(l) = \{ t\bar{v} \mid t \in \mathbb{R} \}. \)
and $T_p \circ T_{\mathbf{v}} \circ T_{-p}$ is translation in the
direction $\mathbf{v}$, while $T_{-p} \circ p \circ T_{-p}$ is reflection
across the line $L = \{p + t\mathbf{v} \mid t \in \mathbb{R}\}$. Now

$$\circ(T_{-p} \circ p \circ T_{-p}) \circ (T_{p} \circ T_{\mathbf{v}} \circ T_{-p})$$

$$= T_{p} \circ p \circ T_{\mathbf{v}} \circ T_{-p} \quad \text{(since $T_{-p} \circ T_{-p} = 2I$)}$$

$$= T_{p} \circ T_{\mathbf{v}} \circ p \circ T_{-p} \quad \text{by our "p=0" argument above}$$

$$= (T_{p} \circ T_{\mathbf{v}} \circ T_{-p}) \circ (T_{p} \circ p \circ T_{-p})$$.

So, setting $r$ to be reflection across $L$ and $T$
to be translation by $\mathbf{v}$, we get

$$r \circ T = T \circ r.$$
Our argument in 5.3.6 already proved

\[ \text{Tor}_2 = \text{Tor}_0 T. \quad \text{Since } \text{Tor}_2 = \text{id}, \]

we get

\[ \text{Tor}_2 \circ \text{Tor}_0 = \text{Tor}_0 \circ \text{id} = \text{id} \circ T = T, \]

as claimed.