Given line $l$ and point $A$ not on $l$, copy $\angle CBA$ to $A$ to construct a line parallel to $l$. We may do this using Proposition I-23 of Euclid:

They are parallel then by Proposition I-27. Suppose $t$ is another line through $A$ not equaling $n$:

Then the interior angles between $\overline{AB}$ and $t$ (that is, on the same side of $t$ as $l$) cannot equal those between $\overline{AB}$ and $n$; since each pair of interior angles (the two between $\overline{AB}$ and $t$ and the two between $\overline{AB}$ and $n$) must add to $180^\circ$, one of the angles between $\overline{AB}$ and $t$ must be smaller than the corresponding angle between $\overline{AB}$ and $n$, the interior angles between $\overline{AB}$ and $t$ and between $\overline{AB}$ and $l$ add to less than $180^\circ$, and so Euclid's 5th Postulate implies that $t$ and $l$ must intersect.
As in the statement of the problem, suppose \( \angle CBA + \angle BAD < 180^\circ \). Copy \( \angle CBA \) to \( A \) to get \( \angle EAF \). Then \( \angle EAF + \angle EAB = 180^\circ \) by Proposition I-13 of Euclid. Now \( \angle EAF = \angle CBA \) by construction, so \( \angle BAD < \angle BAE \); thus the lines \( n \) and \( t \) cannot be equal. As in problem 2.1.5, Proposition I-27 of Euclid implies \( n \) is parallel to \( l \), so Playfair implies that \( t \) is not parallel to \( l \); so \( t \) and \( l \) must intersect. We need to show they intersect on the same side of \( m \) as \( D \) and \( C \).

Suppose not—say, \( l \) and \( t \) intersect at a point \( G \) on the other side of \( m \) from \( C \). Then by the Exterior Angle Theorem, \( \angle ABC \) is greater than \( \angle GAB \). As we saw, \( \angle ABC + \angle BAD < 180^\circ \), so \( \angle GAB + \angle BAD < 180^\circ \).

But this contradicts Proposition I-13 along the line \( t \). So they must intersect on the other side, proving Euclid's Fifth Postulate.
2.1.7. Suppose Playfair's True, \( l \) and \( n \) are parallel, and \( m \) is perpendicular to \( l \); let \( A \) be a point of intersection of \( n \) and \( m \). By Euclid's Prop. I-11, we can draw a perpendicular through \( A \) to \( m \); call it \( k \):

Now by Euclid's Prop. I-27, \( k \) and \( n \) are parallel. Then Playfair implies that \( k = n \), so \( n \) is perpendicular to \( m \).

Conversely, suppose that whenever a line is perpendicular to one of two parallel lines, it must be perpendicular to the other. Suppose \( l \) is a line and \( A \) is a point not on \( l \). By Euclid's Prop. I-12, there is a line through \( A \) perpendicular to \( l \); call it \( m \): 

Now, by Euclid's Prop. I-11, there is a line through \( A \) perpendicular to \( m \), call it \( k \):

By Prop. I-27, \( k \) and \( l \) are parallel. This proves the existence in Playfair. Now if \( n \) is any other line through \( A \) parallel to \( l \), by our hypothesis \( n \) must
be perpendicular to $m$. So $n$ and $k$ both pass through $A$ and are perpendicular to $m$, hence they are equal. This proves uniqueness in Playfair.

Remark: Here's a "proof" that if $n$ and $k$ pass through a point $A$ on line $m$ and are perpendicular to $m$, they are equal; take one ray from each line:

Since $n$ and $k$ are both perpendicular to $m$, the angles are both right angles, so by Prop. 1-14 of Euclid, the rays form a line.

Euclid's Second Postulate should then say that, given a ray, it can be extended uniquely to a line.

[This is a shortcoming in Euclid—compare to Hilbert's Axiom I-2 on p. 445.]
2.2.7. Suppose a triangle has two angles congruent:

\[ \angle ABC = \angle ACB. \]

So \( \triangle ABC \) is congruent to \( \triangle ACB \) — that is, congruent in a way that interchanges \( B \) and \( C \).

Well,

by hypothesis \( \angle ABC = \angle ACB \), and
\( \angle ACB = \angle ABC \) (see labelling);
also side \( BC \) equals side \( CB \) evidently. So
by ASA, the two triangles are congruent; hence side \( AB \) is congruent to side \( AC \), proving that \( \triangle ABC \) is isosceles.