(1) Use a double integral and an appropriate choice of coordinates to compute the area of the region $\mathcal{R}$ bounded by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$ 

(2) Find the center of mass of the lamina bounded by $y = 0$ and

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$ 

(3) Find the area of the region bounded by $y = x$, $y = 2x$, $xy = 1$, $xy = 2$. 

(4) Evaluate

$$\int \int_{\mathcal{R}} \exp \left( \frac{x - y}{x + y} \right) \, dA$$

where $\mathcal{R}$ is bounded by $x = 0$, $y = 0$ and $x + y = 1$ (hint: look at the integrand for a choice of coordinate change). 

(5) Determinants of $2 \times 2$ matrices are related via the following formula

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} p & q \\ r & s \end{vmatrix} = \begin{vmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{vmatrix}.$$ 

Use this and the Chain Rule show that (for any coordinate transformation $(x, y) \leftrightarrow (u, v)$)

$$\frac{\partial(x, y)}{\partial(u, v)} \frac{\partial(u, v)}{\partial(x, y)} = 1.$$ 

(6) Let $\mathcal{R}$ be the region bounded by the curves $xy = 1$, $xy = 3$, $x^2 - y^2 = 1$, and $x^2 - y^2 = 4$. Compute

$$\int \int_{\mathcal{R}} x^2 + y^2 \, dA.$$
Hints: Choose appropriate coordinates. What is \((x^2 + y^2)^2\) in these new coordinates? Use the result of the previous problem.