(1) Compute the gradients of the following functions.
(a) \( f(x, y) = \sqrt{9 - x^2 - y^2} \)
(b) \( f(x, y) = \tan^{-1}(y/x) \).

(2) Let \( \vec{u} \) and \( \vec{v} \) be unit vectors in the plane and \( c \) a fixed real number. For differentiable functions \( f(x, y) \) and \( g(x, y) \) show
(a) \( D_{\vec{u}+\vec{v}}f = D_{\vec{u}}f + D_{\vec{v}}f \),
(b) \( D_{c\vec{u}}f = cD_{\vec{u}}f \), and
(c) \( D_{\vec{u}}(fg) = (D_{\vec{u}}f)g + f(D_{\vec{u}}g) \).

(3) In which unit direction does the function \( f(x, y, z) = z e^{xy} \) increase fastest at the point \( P(0, 2, 1) \)?

(4) Find an equation for the tangent plane \( \mathcal{T} \) at any point \((a, b, c)\) on the surface \( z = x^2 + y^2 \). Write an equation for the line formed by the intersection of \( \mathcal{T} \) and the \( xy\)-plane. Show that this line is tangent to the circle in the \( xy\)-plane centered at the origin with radius \( \sqrt{(a^2 + b^2)/4} \).

(5) Show that \( D_{\vec{v}}f(x, y) = \pm \frac{\partial f}{\partial x}(x, y) \) for a vector \( \vec{v} \) parallel to \( \vec{i} \).

(6) The directional derivative of a function of two variables is (generally) again a function of two variables...ready to be differentiated in another direction. In particular, for two unit vectors \( \vec{u} \) and \( \vec{v} \), we define
\[
D^2_{\vec{u}\vec{v}}f = D_{\vec{u}}(D_{\vec{v}}f).
\]
Compute \( D^2_{\vec{u}\vec{v}}f \) and \( D^2_{\vec{v}\vec{u}}f \) where \( f(x, y) = x^2y \), \( \vec{u} = \langle -1, 1 \rangle \) and \( \vec{v} = \langle 2, 1 \rangle \). Notice anything? As a challenge, try to show your observation holds for arbitrary choices of unit vectors and functions.
(7) Suppose $f(x, y)$ is a function with continuous second partial derivatives. Let
\[ \nabla f(x, y) = \langle P(x, y), Q(x, y) \rangle. \]
What must be true about the derivatives of $P(x, y)$ and $Q(x, y)$?