Math 241, Fall 2012

Lecture 1

Calculus: The study of quantities that vary over space/time. Such quantities are encoded in functions.

Example: \( H(t) \) heart rate of Professor Newman at time \( t \).

What do we study about such things in calculus?

- (instantaneous) rate of change

\[
\begin{align*}
    \text{Heart Rate (bpm)} \\
    \text{H} \\
    \text{class} \\
    \text{time} \\
    \text{t}
\end{align*}
\]

- "total" over a period of time.

\[
\text{area} \int_{a}^{b} H(t) \, dt
\]

total \# of heart beats in time period
Fundamental Theorem of Calculus

If \( f(t) = F'(t) \), then

\[
\int_{a}^{b} f(t) \, dt = F(b) - F(a)
\]

In the Example Since \( H(t) = B'(t) \), where

\( B(t) = \# \text{ of times Nancy's heart has beaten from the beginning of time until time } t \), get

\[
\int_{a}^{b} H(t) \, dt = B(b) - B(a) = \# \text{ of beats between time } a \text{ and time } b.
\]

Typical Real-World Problems Usually quantities depend on much more information. And quantities themselves can be more complex!

Example Temperature at a point on map at time \( t \):

\( T(\text{latitude}, \text{longitude}, t) = \text{function of three inputs} \)

\[ \text{[latitude too?]} \]

Example particle moving in space is described by its position function \( \vec{r}(t) \) - at time \( t \), \( \vec{r}(t) \) is position.

Ex in Plane

\( \vec{r}(t) = (x(t), y(t)) \)

\[ \text{in space} \]

\( \vec{r}(t) = (x(t), y(t), z(t)) \)
Common in Physics record both position and momentum
\[
\mathbf{r}(t) = (q_1(t), q_2(t), q_3(t), p_1(t), p_2(t), p_3(t)) \in \text{ phase space.}
\]

I can't draw it! [What about millions of particles?]

Main Theme of the Course Apply the techniques of calculus to the geometry of space!

[Emphasis on understanding geometry.]

Outline
1) vectors and geometry of n-dimensional space.
2) basics of functions of several variables and with several outputs.
3) optimization (min/max)
4) integration
5) curves and surfaces in $\mathbb{R}^3$

(6) COURSE HIGHLIGHT Generalize FTC, Conservation laws!