Math 428, Homework 1, due Friday, August 25

**Problem 1.** Describe the maximal ideals in \( \mathbb{C}[x] \); describe the prime ideals in \( \mathbb{C}[x] \). Justify your answer.

**Problem 2.** Describe the prime ideals in \( \mathbb{R}[x] \). Justify your answer.

**Problem 3.** Describe the prime ideals in \( \mathbb{C}[x]/(x^2 - 1) \). Justify your answer.

**Problem 4.** Describe the prime ideals in \( \mathbb{Z} \). Justify your answer.

**Problem 5.** Let \( M \) denote the ring of \( 2 \times 2 \) complex matrices with usual matrix addition and multiplication (recall that this ring is not commutative). Let
\[
X = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad Y = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.
\]
Define functions \( \phi : \mathbb{C}[x] \to M \) and \( \psi : \mathbb{C}[y] \to M \) by \( \phi(f) = f(X) \) and \( \psi(g) = g(Y) \). Prove that \( \phi \) and \( \psi \) are ring homomorphisms. Compute the kernels of \( \phi \) and \( \psi \). What are the prime ideals of the images of \( \phi \) and \( \psi \)?