Enumerative Geometry, Mirror Symmetry and Physics

Lara Anderson

All fibrations in CICY threefolds

Abstract: I will describe a recent effort to systematically enumerate all genus one fibrations in the class of 7890 Calabi-Yau manifolds defined as complete intersections in products of projective spaces (so-called CICY threefolds). This survey is a complete classification that depends only on topology and improves upon past approaches that probed only a particular algebraic form of the manifold. I will also describe K3-fibrations and so-called "nested" fibration structures (for example, K3 fibrations in which the generic fiber can in turn can be elliptically fibered in many distinct ways).

Per Berglund

A generalized construction of Calabi-Yau manifolds and mirror symmetry

Abstract: We extend the construction of Calabi-Yau manifolds to hypersurfaces in non-Fano toric varieties. The associated non-reflexive polytopes provide a generalization of Batyrev's original work, allowing us to construct new pairs of mirror manifolds. In particular, this allows us to find new K3-fibered Calabi-Yau manifolds, relevant for type IIA/heterotic duality in $d = 4, N = 2$, string compactifications.

Jim Bryan

Banana manifolds, elliptic genera, and Katz invariants

Abstract: I will discuss Donaldson-Thomas and Katz invariants of the banana manifold, a compact Calabi-Yau threefold X fibered by Abelian surfaces whose singular fibers contain certain configurations of rational curves called bananas. We prove that the Donaldson-Thomas partition function of X, for fiberwise curve classes, is given by a beautiful four variable infinite product expansion. Amazingly, this partition function, after a change of variables, coincides with the generating function for the equivariant elliptic genera of the Hilbert scheme of points in the plane. We also discuss the Katz invariants, a mathematical version of genus 0 BPS state counts, for this geometry. The Katz invariants of banana configurations are governed by a well known theta function, which we can see arise from the banana geometry via a trick dating back to a collaboration with Katz in the late 90s.

Andrei Caldararu

Computing a categorical Gromov-Witten invariant

Abstract: In his 2005 paper "The Gromov-Witten potential associated to a TCFT" Kevin Costello described a procedure for recovering an analogue of the Gromov-Witten potential directly out of a cyclic A-inifinity algebra or category. Applying his construction to the derived category of sheaves of a complex projective variety provides a definition of higher genus B-model Gromov-Witten invariants, independent of the BCOV formalism. This has several advantages.

Due to the categorical invariance of these invariants, categorical mirror symmetry automatically implies classical mirror symmetry to all genera. Also, the construction can be applied to other categories like categories of matrix factorization, giving a direct definition of FJRW invariants, for example.

In my talk I shall describe the details of the computation (joint with Junwu Tu) of the invariant, at $g = 1, n = 1$, for elliptic curves. The result agrees with the predictions of mirror symmetry, matching classical calculations of Dijkgraaf. It is the first non-trivial computation of a categorical Gromov-Witten invariant.

Jinwon Choi

Genus zero log BPS numbers for del Pezzo surfaces

Abstract: We explore the enumerative relationship between local and log Calabi-Yau geometries for del Pezzo surfaces. Genus zero log BPS numbers for del Pezzo surfaces are defined from the log Calabi-Yau geometry of the surface with a smooth anticanonical divisor. We propose their conjectural relationship to the genus zero local BPS state counts. This talk is based on joint work with Michel van Garrel, Sheldon Katz and Nobuyoshi Takahashi.

Herb Clemens

A global model for F-theory-Heterotic duality with Wilson line symmetry-breaking and $\mathbb{Z}/4\mathbb{Z}$ R-symmetry

Abstract: String theory proposes $SU(5)$-symmetry of the four fundamental forces during the first few nano-seconds after the Big Bang followed by breaking to the $SU(3) \times SU(2) \times U(1)$-symmetry of the Standard Model via a Wilson line when energy levels dropped below 10^{16} GeV as the cosmos continued to expand. Based on joint work with David Morrison, Tony Pantev, Stuart Raby and Sakura Schafer-Nameki, I will propose a global mathematical model for that evolution consistent with the theoretical and experimental constraints imposed by physical theory.

David Cox

Moment maps of toric varieties, linear precision, and maximum likelihood degree one

Abstract: The quotient construction of the toric variety of a lattice polytope gives a canonically defined moment map. Other moment maps come from projective embeddings of the toric variety that use the ample line bundle determined by the polytope. It is natural to ask if the quotient moment map is one of these projective moment maps. My lecture will explain how this relates to linear precision in geometric modeling and and maximum likelihood degree one in algebraic statistics. This is joint work with Patrick Clarke of Drexel University.

Tudor Dimofte

G-actions and Koszul duality in supersymmetric quantum mechanics

Abstract: I will revisit the structure of G-symmetry in supersymmetric quantum mechanics. When G is gauged, it is well known to lead to equivariance. When G is a flavor symmetry, it leads to a less-well-known algebra of fermionic operators

acting on the Hilbert space, associated with the homology of G. Mathematically, these two structures were related by a version of Koszul duality in the work of Goresky-Kottwitz-MacPherson. The "duality" has a natural physical origin in two-dimensional A-twisted gauge theory and its category of boundary conditions.This provides one of a growing number of examples of Koszul duality in physics. (Joint work with David Ben-Zvi and Andy Neitzke.)

Ron Donagi

Geometric Langlands and Hodge theory

Min-xin Huang

Refined BPS invariants of 6d SCFTs from anomalies and modularity

Abstract: F-theory compactifications on appropriate local elliptic Calabi-Yau manifolds engineer six dimensional superconformal field theories and their mass deformations. The partition function $Z_{\rm top}$ of the refined topological string on these geometries captures the particle BPS spectrum of this class of theories compactified on a circle. Organizing $Z_{\rm top}$ in terms of contributions Z_β at base degree β of the elliptic fibration, we find that these, up to a multiplier system, are meromorphic Jacobi forms of weight zero with modular parameter the Kaehler class of the elliptic fiber and elliptic parameters the couplings and mass parameters. The indices with regard to the multiple elliptic parameters are fixed by the refined holomorphic anomaly equations, which we show to be completely determined from knowledge of the chiral anomaly of the corresponding SCFT. We express Z_β as a quotient of weak Jacobi forms, with a universal denominator inspired by its pole structure as suggested by the form of $Z_{\rm top}$ in terms of 5d BPS numbers. The numerator is determined by modularity up to a finite number of coefficients, which we prove to be fixed uniquely by imposing vanishing conditions on 5d BPS numbers as boundary conditions. We demonstrate the feasibility of our approach with many examples, in particular solving the E-string and M-string theories including mass deformations, as well as theories constructed as chains of these. We make contact with previous work by showing that spurious singularities are cancelled when the partition function is written in the form advocated here. Finally, we use the BPS invariants of the E-string thus obtained to test a generalization of the Goettsche-Nakajima-Yoshioka K-theoretic blowup equation, as inspired by the Grassi-Hatsuda-Marino conjecture, to generic local Calabi-Yau threefolds.

Hans Jockers

M-theory on twisted connected sum G2-manifolds

Abstract: I review the dimensional reduction of M-theory on G2-manifolds that are constructed by Kovalev's twisted connected sum by gluing suitable pairs of asymptotically cylindrical Calabi-Yau threefolds augmented with a circle S^1. The low energy effective theory exhibits in a certain limit gauge theory sectors with extended supersymmetry. Suitable singular asymptotically cylindrical Calabi-Yau threefolds lead to non-Abelian gauge symmetries with matter in the discussed M-theory context. Studying the obtained low-energy effective gauge

theories, I describe geometric transitions among G2-manifolds and also obtain new explicit examples of G2-manifolds. This talk is based upon joint work with Thaisa Guio, Albrecht Klemm, and Hung-Yu Yeh.

Shamit Kachru

Refined BPS counts and automorphic jumping phenomena

Abstract: I describe a new, more refined counting function for BPS states in 5d and 4d compactifications of M theory and string theory, the Hodge-elliptic genus. This function is not an index, and its jumping behavior in moduli space is automorphic, making contact with ideas of Hirzebruch-Zagier and Kudla-Millson.

Albrecht Klemm

D-brane masses and the motivic Hodge conjecture

Abstract: We calculate the masses of the D2 and D4 brane at the conifold of one parameter Calabi-Yau spaces, using the motivic Hodge conjecture. This gives evidence for the latter in a new context.

Chiu-Chu Melissa Liu

Multiple covers of an orbidisk and orbifold GW/DT correspondence

Abstract: Using localization, Sheldon Katz and the speaker expressed open GW invariants counting multiple covers of a generic disc bounding a special Lagrangian submanifold in a Calabi-Yau 3-fold in terms of certain cubic Hodge integrals. This leads to a formula of such Hodge integrals conjectured by Marino-Vafa, which is equivalent to the GW/DT correspondence for the framed 1-leg topological vertex. After a brief review of relevant early results, I will describe more recent results on multiple covers of an orbidisc bounding a special Lagrangian sub-orbifold in a Calabi-Yau 3-orbifold, and orbifold GW/DT correspondence, based on work of Dustin Ross, Zhengyu Zong, Ross-Zong, Zijun Zhou, and Zhou-Zong.

Davesh Maulik

Gopakumar-Vafa invariants via vanishing cycles

Abstract: Given a Calabi-Yau threefold X,I'll explain a proposal for defining Gopakumar-Vafa invariants of X via sheaves of vanishing cycles, building on earlier approaches of Kiem-Li and Hosono-Saito-Takahashi. Conjecturally, these should agree with the invariants as defined by stable maps. I will also explain how to prove the conjectural correspondence for irreducible curves on local surfaces. This is joint work with Yukinobu Toda.

David Morrison

Higher Chow groups, van Geemen lines, and mirror symmetry for open strings

Abstract: This is a report on work in progress with Hans Jockers and Johannes Walcher.

Andy Neitzke

BPS states in the E_6 Minahan-Nemeschansky theory

Abstract: I will describe a recent computation of BPS state counts in a certain 4d N=2 supersymmetric field theory, the "Minanan-Nemeschansky theory with E_6 flavor symmetry." These BPS state counts are conjecturally equal to generalized Donaldson-Thomas invariants for an appropriate CY3 category. The result has several mysterious features, including some algebraic equations obeyed by the generating functions of the counts, and a relation to Gopakumar-Vafa invariants of a Calabi-Yau threefold. This is a report of joint work with Lotte Hollands.

Tom Nevins

Compactifications, cohomology, and categories associated to moduli spaces

Abstract: Many moduli spaces (for example, of instantons or vacua) appearing in supersymmetric field theories admit modular compactifications with especially good properties. I will outline a general strategy, building on pioneering work of Beauville, Markman, and others, for describing generators of topological and and categorical invariants associated to such moduli spaces. To illustrate, I will describe joint work with K. McGerty that applies the strategy to prove "Kirwan surjectivity" for Nakajima quiver varieties. Work in progress with McGerty derives general statements (of which the quiver variety results are a special case) applicable to many $3d$ $N = 4$ settings.

Rahul Pandharipande

Cohomological field theories from local curves

Abstract: I will explain work with H.H. Tseng on the higher genus Gromov-Witten theory of the Hilbert scheme of points of the plane and its relationship to local curve theories.

Yongbin Ruan

Computing higher genus Gromov-Witten of the quintic

Abstract: In every subject of mathematics, there are a few guiding problems which are usually very difficult and yet inspire much of activities. The computation of higher genus Gromov-Witten invariants of quintic (more generally compact Calabi-Yau manifolds) is such an example. It inspired the much developments in mathematics during last fifteen years such as the modularity properties, FJRW-theory and recent mathematical GLSM-theory. In the talk, I will survey these development and some recent outlook.

Sakura Schafer-Nameki

Gauge theories and elliptic fibrations

Eric Sharpe

Quantum sheaf cohomology and (0,2) Landau-Ginzburg mirrors

Abstract: In this talk I will outline a generalization of mirror symmetry known as (0,2) mirror symmetry, involving spaces together with holomorphic vector bundles satisfying certain consistency conditions. I will begin by briefly reviewing "quantum sheaf cohomology," a generalization of quantum cohomology that has been worked out for toric varieties and Grassmannians with tangent bundle

deformations, in work done in collaboration with S. Katz, R. Donagi, Z. Lu, J. Guo, and others. I will then discuss analogues of Toda duals, specifically, B/2-twisted (0,2) Landau-Ginzburg models whose correlation functions encode quantum sheaf cohomology of their A/2 model duals.

Artan Sheshmani

Nested Hilbert schemes, local Donaldson-Thomas theory, Vafa-Witten and Seiberg-Witten invariants

Abstract: We report on the recent rigorous and general construction of the deformation-obstruction theories and virtual fundamental classes of nested (flag) Hilbert scheme of one dimensional subschemes of a smooth projective algebraic surface. This construction will provide one with a general framework to compute a large class of already known invariants, such as Poincare invariants of Okonek et al, or the reduced local invariants of Kool and Thomas in the context of their local surface theory. We show how to compute the generating series of deformation invariants associated to the nested Hilbert schemes, and via exploiting the properties of vertex operators, prove that in some cases they are given by modular forms. We finally establish a connection between the Vafa-Witten invariants of local-surface threefolds (recently analyzed also by Tanaka and Thomas) and such nested Hilbert schemes. This construction (via applying Mochizuki's wall-crossing techniques) enables one to obtain a relations between the generating series of Seiberg-Witten invariants of the surface, the Vafa-Witten invariants and some modular forms. This is joint work with Amin Gholampour and Shing-Tung Yau following `arXiv:1701.08902` and `arXiv:1701.08899`.

Arnav Tripathy

BPS algebras, generalized Kac-Moody algebras, and enumerative geometry