1. A population $P$ of bacteria starts with 100 bacteria and triples in size every half hour.

(a) How many bacteria are there after 2 hours?

$$P(2) = 8100$$

(b) How many bacteria are there after $t$ hours?

$$P(t) = 100 \cdot 3^{2t}$$

(c) At what time will there be 1000 bacteria? *Give an exact answer involving logs.*

$$t = \frac{1}{2} \log_3 10$$

$$t = \frac{1}{2} \log_3 10$$
2. Find all values of $x$ in the interval $[0, 2\pi]$ that satisfy the equation

$$\csc^2 x = 2.$$

Give your answer in radians; it should not involve any inverse trig functions.

$$\csc^2 x = 2 \Rightarrow \sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$$

$x \in [0, 2\pi]$

- $\sin x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$
- $\sin x = -\frac{\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

3. Find the exact value of $L$, where

$$L = \frac{1}{3} \log_3 8 - \log_3 90 + \log_3 1 - \log_3 (\frac{1}{3})$$

There should not be any logs in your answer.

$$L = \log_3 8^{\frac{1}{3}} + \log_3 \frac{1}{90} + \log_3 6 + \log_3 5$$

$$L = \log_3 \left( \frac{2 \cdot 5}{90} \right) = \log_3 \left( \frac{1}{9} \right) = -2$$

$$L = -2$$

4. If $\sec \theta = \frac{5}{4}$ and $0 \leq \theta \leq \frac{\pi}{2}$, use a trig identity to evaluate $\sin(2\theta)$

$$\cos \theta = \frac{4}{5}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{3}{5} \right) \left( \frac{4}{5} \right) = \frac{24}{25}$$

$$\sin(2\theta) = \frac{24}{25}$$