Math 241 (sections DD7 and DD2) Quiz 9 Spring 2012

Name ____________________________

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 20 minutes for this quiz.

1. (8 points) Continuation of an Example from Class: Let $R$ be the region bounded by the curves $x^2 + y^2 = 2$, $y = 1/2$, and $y = -1/2$.

(a) Let $v = y$ and $u = \frac{x}{\sqrt{2-y^2}}$. Draw the region $R$ in the $u$, $v$ coordinate system.

(b) Solve the equations for $x$ as a function of $u$ and $v$. Similarly, obtain $y$ as a function of $u$ and $v$, i.e. What are $x(u,v)$ and $y(u,v)$?

\[ x = u \sqrt{2-y^2} \]
\[ y = v \]

(c) What is $\left| \frac{\partial(x,y)}{\partial(u,v)} \right|$?

\[
\begin{bmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{bmatrix} = \begin{bmatrix}
\sqrt{2-v^2} & -uv \\
0 & \sqrt{2-v^2}
\end{bmatrix} = \sqrt{2-v^2}.
\]
(d) Write $\int \int_R \frac{1}{\sqrt{2-y^2}} \, dy \, dx$ in the $u,v$ coordinate system.

$$= \frac{1}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\sqrt{2-1^2}}^{\sqrt{2-1^2}} \frac{1}{\sqrt{2-v^2}} \, du \, dv = \frac{\sqrt{2}}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{2} \, du \, dv.$$ 

2. Orthogonal Vector Fields: Let $f(x,y) = x^2 + y^2$. Let $F(x,y) = (y,-x)$. Sketch the vector field $\nabla f(x,y)$ OR sketch the vector field $F(x,y)$.

i) $\nabla f$ is orthogonal to circles centered at the origin.

OR

ii) $F(xy) = (y,-x)$

The easiest way to approach this is to note that $F(xy) \cdot \nabla f(xy) = 0$ for any $(xy)$ pair. So, all vectors you draw should be tangent to circles. You just need to decide where they point.

Clockwise vs. counter-clockwise.