Math 241 (section DD1)  
Quiz 4  
Spring 2012

Name ________________________________

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (1 point) $\int_0^\pi \int_0^\pi \int_0^2 \rho^2 \sin \phi d\rho d\phi d\phi$ Gives the volume of 
   a) half a sphere of radius 2 
   b) one eighth of a sphere of radius 2 
   c) one quarter of a sphere of radius 2.

2. (3 point) Change $\int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-z^2}}^{\sqrt{3-z^2}} \int_1^{4-z^2-y^2} \frac{1}{r^2} dz dy dx$ to cylindrical coordinates. 
   (Do Not Evaluate)

   $\begin{align*}
   \chi &= \sqrt{\cos \phi} \\
   \gamma &= r \sin \phi
   \end{align*}$

\[
\int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-z^2}}^{\sqrt{3-z^2}} \int_1^{4-z^2-y^2} \frac{1}{r^2} dz dy dx
\]
3. (3 points) Change \( \int_0^3 \int_{\sqrt{9-x^2}}^{\sqrt{9-y^2}} \int_{\sqrt{9-y^2-z^2}}^{\sqrt{9-y^2}} x^2 \, dx \, dy \, dz \) to spherical coordinates.
(Do Not Evaluate)

Top half of a sphere of radius 3

\[
\frac{3}{2} \pi \int_0^{\frac{\pi}{2}} \int_0^\pi (\rho^2 \sin^2 \phi) \rho^2 \sin \phi \, d\phi \, d\theta \, d\rho
\]

4. (3 points) Set up the integral \( \iiint_E x^2 \, dV \), where \( E \) is bounded by the \( xy \)-plane and the hemispheres \( z = -\sqrt{9-x^2-y^2} \) and \( z = -\sqrt{9-x^2-y^2} \).
(Do Not Evaluate)

Bottom half a sphere of radius 3 with removed bottom half of a sphere radius 2

\[
\frac{\pi}{2} \int_0^{\frac{\pi}{2}} \int_0^\pi (\rho^2 \sin^2 \phi \cos^2 \theta) \rho^2 \sin \phi \, d\phi \, d\theta \, d\phi
\]