No calculators allowed.
Show sufficient work to justify each answer.
You have 15 minutes for this quiz.

1. (3 points) Evaluate the double integral: \( \int_0^1 \int_x^1 e^{x+y} dy \, dx \). (Hint: Change the order of integration).

\[ i) \text{ } x \text{ varies from } 0 \text{ to } 1. \]
\[ ii) \text{ } \text{For a fixed } x \text{, } y \text{ varies from } x \text{ to } 1. \]

If we integrate with respect to \( x \) first, then we are fixing (temporarily) \( y \).

Hence, the integral becomes:

\[ \int_0^1 \int_x^1 e^{x+y} \, dy \, dx = \int_0^1 \left[ \frac{e^{x+y}}{x} \right]_0^y \, dy = \int_0^1 \frac{e^y}{y} \, dy = \int_0^1 [e - 1] \, dy \]

\[ = (e-1) \int_0^1 y \, dy = \frac{1}{2} (e-1). \]

\textbf{Note:} It was a common mistake to write \( \int e^{xy} \, dx = \frac{1}{y} e^{xy} \), in which case the integration yields \( \infty \). Thus, it was necessary to mention that some integrals can yield \( \infty \) since this could have been somewhat confusing.
2. (4 points) In the $xy$-plane, the region $R$ is bounded by the curves $y = x + 2$, $y = 2 - x$, and $x = -2$. Sketch the region and determine the volume of the solid bounded above by $f(x, y) = \frac{y}{1+x^2}$ and below by the region $R$.

\[
\int_{-2}^{0} \int_{2-x}^{2-x} \frac{y}{1+x^2} \, dy \, dx = \frac{1}{2} \int_{-2}^{0} \left[ \frac{(2-x)^2 - (x+2)^2}{1+x^2} \right] \, dx \\
= \frac{1}{2} \int_{-2}^{0} -\frac{8x}{1+x^2} \, dx \\
= -2 \int_{-2}^{0} \frac{2x}{1+x^2} \, dx \\
= -2 \left[ \ln(1+x^2) \right]_{-2}^{0} = 2 \ln(5).
\]

3. (3 points) Use a double integral to compute the area of the region bounded by the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ between $x = \pi/4$ and $x = 5\pi/4$.

i) $\cos(x) = \sin(x)$ is true when $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$ on $[0, 2\pi]$.

ii) $\sin(x) \geq \cos(x)$ on $\left[ \frac{\pi}{4}, \frac{5\pi}{4} \right]$ since

$\sin\left( \frac{\pi}{4} \right) > \cos\left( \frac{\pi}{4} \right)$ and $\sin(x) - \cos(x) \neq 0$ on $(\frac{\pi}{4}, \frac{5\pi}{4})$.

\[
A = \int_{\pi/4}^{5\pi/4} \int_{\cos(x)}^{\sin(x)} (1) \, dy \, dx = \int_{\pi/4}^{5\pi/4} \left[ \sin(x) - \cos(x) \right] \, dx \\
= -\left[ \cos(x) + \sin(x) \right] \bigg|_{\pi/4}^{5\pi/4} = -\left[ -\sqrt{2} - \sqrt{2} \right] = 2\sqrt{2}.
\]