Name ______________________________________

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

Do one out of the two 1 point problems

1. (1 point) True or False \(< \cos(3t), \sin(3t) > \) and \(< 3\cos(t), 3\sin(t) > \) generate the same curve.

2. (1 point) Find the domain of the vector function \( r(t) = \sqrt{16 - t^2}, \ln(t + 3), \frac{1}{1-t} \) 
\((-\infty, 1) \cup (1, 4]\)

Do all three

1. (3 points) Find the parametric equation for the tangent line to the curve with given parametric equations at the specified point.
\( x = 2 + \ln(t), \ y = 3t^3, \ z = 3t + 1; \ (2,3,4) \quad t=1 \)

\( r'(t) = \langle \frac{1}{t}, 9t^2, 3 \rangle \)
\( r'(1) = \langle 1, 9, 3 \rangle \)

\[ t = \left\{\begin{array}{l}
\begin{align*}
x &= 2 + t \\
y &= 3 + 9t \\
z &= 4 + 3t
\end{align*}
\end{array}\right. \]
2. (3 points) Find the length of the curve given by \( \mathbf{r}(t) = (2, 2t^2, t^3) \) for \( 1 \leq t \leq 2 \).

\[ \mathbf{r}'(t) = \langle 0, 4t, 3t^2 \rangle \]

\[ |\mathbf{r}'(t)| = \sqrt{16 + 9t^4} = 4 + \sqrt{9t^4} \]

\[ L = \int_{1}^{2} \sqrt{16 + 9t^4} \, dt \]

\[ u = 16 + 9t^4 \]

\[ \frac{du}{dt} = 36t^3 \]

\[ L = \int_{25}^{52} \frac{\sqrt{u}}{18} \, du \]

\[ = \frac{52}{18} \left( \frac{2}{3} \right) \left( 2 \frac{2}{3} - 25 \frac{2}{3} \right) = \frac{1}{27} \left( 52 - 125 \right) \times \frac{1}{27} \left( 2 \frac{2}{3}, 9 \frac{7}{9} \right) \]

3. (3 points) Let \( \mathbf{r}(t) = \langle e^{2t}\cos(2t), 5, e^{2t}\sin(2t) \rangle \), find \( |\mathbf{r}'(t)| \). Simplify final solution.

\[ \mathbf{r}'(t) = \langle 2e^{2t}\cos(2t) - 2e^{2t}\sin(2t), 0, 2e^{2t}\sin(2t) + 2e^{2t}\cos(2t) \rangle \]

\[ |\mathbf{r}'(t)| = \sqrt{4e^{4t}(\cos^2(2t) - \sin^2(2t))^2 + 4e^{4t}(\sin^2(2t) + \cos^2(2t))^2} \]

\[ = \sqrt{4e^{4t}(\cos^2(2t) - 2\cos(2t)\sin(2t) + \sin^2(2t)) + 4e^{4t}(\cos^2(2t) + 2\cos(2t)\sin(2t) + \sin^2(2t))} \]

\[ = \sqrt{4e^{4t}(\cos^2(2t) + \sin^2(2t))} \]

\[ = \sqrt{4e^{4t} + 4e^{4t}} \]

\[ = \sqrt{8e^{4t}} \]

\[ = 2\sqrt{2}e^{2t} \]