Name: Solutions

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 points) Find the area of the triangle with vertices $P(1, 1, 1)$, $Q(2, -1, 3)$ and $R(3, 4, 3)$.

   \[ \vec{PQ} = \langle 1, -2, 2 \rangle \]
   \[ \vec{PR} = \langle 2, 3, 2 \rangle \]

   \[ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & 2 \end{vmatrix} = -10 \hat{i} + 2 \hat{j} + 7 \hat{k} \]

   \[ |\vec{PQ} \times \vec{PR}| = \sqrt{10^2 + 2^2 + 7^2} = \sqrt{153} \]

   \[ A = \frac{\text{Area of Parallelogram}}{2} = \frac{|\vec{PQ} \times \vec{PR}|}{2} = \frac{\sqrt{153}}{2} \]
2. (4 points) Determine parametric equations for the line which goes through the point \((3, 1, 4)\) and is perpendicular to the plane \(3x + 1y - 4z = 6\).

The normal vector of the plane is \(\hat{n} = \langle 3, 1, -4 \rangle\), so the line \(L\) is parallel to \(\hat{n}\).

\[
L(t) = \langle 3, 1, 4 \rangle + t \langle 3, 1, -4 \rangle = \langle 3 + 3t, 1 + t, 4 - 4t \rangle
\]

\[
\Rightarrow \quad x(t) = 3 + 3t, \quad y(t) = 1 + t, \quad z(t) = 4 - 4t
\]

3. (3 points) Find the distance from the point \((1, 3, 7)\) to the given line \(L\).

\[
L : \begin{cases} x = 3 + 1t \\ y = 4 \\ z = 1 - t \end{cases}
\]

The vector \(\vec{a} = \langle 1, 3, 7 \rangle - \langle 3, 4, 1 \rangle = \langle -2, -1, 6 \rangle\) is parallel to the line, and \(\vec{b} = \langle 1, 0, -1 \rangle\) is a vector from the origin to the line.

For example:

\[
d = \frac{|\vec{a} \times \vec{b}|}{|\vec{b}|} = \frac{|\langle -2, -1, 6 \rangle|}{\sqrt{2}} = \frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3
\]

\[
d = |\vec{a} - \text{proj}_b \vec{a}|\] as well.