1. (4 points) Let \( f(x, y, z) = xy - xz \), and \( C \) be a curve lie in the intersection of two surfaces \( y = x^2 + 1 \) and \( z = x^2 - 1 \) from \((0, 1, -1)\) to \((1, 2, 0)\). Compute the line integral \( \int_C f(x, y, z) \, ds \).

\[
C: \quad R(t) = (t, \ t^2+1, \ t^2-1), \quad 0 \leq t \leq 1.
\]

\[
\int_C f(x, y, z) \, ds = \int_0^1 \left( t (t^2+1) - t (t^2-1) \right) \sqrt{1^2 + (2t)^2 + (2t)^2} \, dt
\]

\[
= \int_0^1 2t \sqrt{1 + 8t^2} \, dt
\]

\[
= \left[ \frac{1}{12} (1 + 8t^2)^{3/2} \right]_0^1
\]

\[
= \frac{1}{12} \left( 9^{3/2} - 1 \right) = \frac{26}{12} = \frac{13}{6}
\]
2. (3 points) Find a function \( f \) on \( \mathbb{R}^3 \) whose gradient is \( \mathbf{F} = (e^x + yz, z^2 + xz, 2yz + xy) \).

\[
\begin{align*}
\frac{\partial f}{\partial x} &= e^x + yz & \Rightarrow & & f = e^x + xyz + h(y, z) & \text{for a function } h.
\frac{\partial f}{\partial y} &= xz + h_y = z^2 + xz & \Rightarrow & & h_y = z^2 & \text{so } h(y, z) = yz^2 + g(z) \\
& & & & \text{for some function } g.
\end{align*}
\]

Then \( f = e^x + xyz + yz^2 + g(z) \)

\[
\begin{align*}
\frac{\partial f}{\partial z} &= xy + 2yz + g'(z) = 2yz + xy & \Rightarrow & & g'(z) = 0 & \text{and then}
\end{align*}
\]

\( g(z) \) is a constant function.

Set \( g(z) = 0 \). Then

\[
\begin{align*}
\text{so } f(x, y, z) = e^x + xyz + yz^2 & \text{ is a such function.}
\end{align*}
\]

3. (3 points) Compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is a line segment in \( \mathbb{R}^3 \) from \((1,0,0)\) to \((0,0,1)\), and \( \mathbf{F} = (e^x + yz, z^2 + xz, 2yz + xy) \).

Since \( \nabla f = \mathbf{F} \), where \( f \) is a function from \#2.

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = f(0,0,1) - f(1,0,0) = 1 - e.
\]

Or, set \( \mathbf{r}(t) = (1-t,0,t), \ 0 \leq t \leq 1 \) is a parametrization for \( C \).

Then

\[
\begin{align*}
\int_0^1 (e^{1-t}, t^2 + t(1-t), 0) \cdot (-1,0,1) \ dt &= \int_0^1 -e^{1-t} \ dt = \left[ e^{1-t} \right]_0^1 = 1 - e.
\end{align*}
\]