1. (4 points) Write T if the statement is true and F if the statement is false. If it is false, then explain why by giving an example.

(1) The union of two simply connected domains is connected.

(2) If a differentiable vector field \( \mathbf{F} \) satisfies \( \int_C \mathbf{F} \cdot d\mathbf{r} = 0 \) for every closed path \( C \) in a connected domain \( D \), then there always exists a function \( f \) such that \( \nabla f = \mathbf{F} \) on \( D \).

(3) If a differentiable vector field \( \mathbf{F} = P\mathbf{i} + Q\mathbf{j} \) satisfies
\[
\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}
\]
on a connected domain \( D \), then \( \mathbf{F} \) is conservative on \( D \).

2. (2 points) Find a function \( f \) such that \( \nabla f = \mathbf{F} \).

\[
\mathbf{F}(x, y) = (\ln y + 2xy^3)\mathbf{i} + (3x^2y^2 + x/y)\mathbf{j}
\]
3. (4 points) Prove that if \( \int_C \mathbf{F} \cdot d\mathbf{r} \) is independent of path in a connected domain \( D \), then
\[
\int_C \mathbf{F} \cdot d\mathbf{r} = 0 \text{ for every closed path } C \text{ in } D.
\]