MATH 234

Test 3

Spring 2018

Circle your TA discussion section.

- BDA, TR 9:00-9:50, Liz Tatum
- BDB, TR 10:00-10:50, Vlad Sadoveanu
- BDC, TR 11:00-11:50, Pedro Mendes de Araujo
- BDD, TR 12:00-12:50, Mike Livesay
- BDE, TR 1:00-1:50, Pedro Mendes de Araujo
- BDF, TR 2:00-2:50, Li Gao
- BDG, TR 3:00-3:50, Li Gao
- BDH, TR 10:00-10:50, Liz Tatum
- CDB, TR 11:00-11:50, Cameron Rudd
- CDC, TR 12:00-12:50, Kexin Jin

- CDD, TR 1:00-1:50, Mike Livesay
- CDE, TR 2:00-2:50, Kexin Jin
- CDF, TR 3:00-3:50, Argen West
- CDG, TR 11:00-11:50, Nachiketa Adhikari
- DDA, TR 9:00-9:50, Vlad Sadoveanu
- DDB, TR 10:00-10:50, Nachiketa Adhikari
- DDC, TR 11:00-11:50, Chris Gartland
- DDD, TR 12:00-12:50, Chris Gartland
- DEE, TR 1:00-1:50, Cameron Rudd
- DDF, TR 2:00-2:50, Argen West

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.

L 1 2 3 4 5 6 7 8 9 10 13 L 1 4 5 6 7 8 9 10 11 12 13 14 L
K 1 2 3 4 5 6 7 8 9 10 11 12 13 K 1 2 3 4 5 6 7 8 9 10 11 12 13 14 K
J 1 2 3 4 5 6 7 8 9 10 11 12 13 J 1 2 3 4 5 6 7 8 9 10 11 12 13 14 J
I 1 2 3 4 5 6 7 8 9 10 11 12 13 I 1 2 3 4 5 6 7 8 9 10 11 12 13 14 I
H 1 2 3 4 5 6 7 8 9 10 11 12 13 H 1 2 3 4 5 6 7 8 9 10 11 12 13 14 H
G 1 2 3 4 5 6 7 8 9 10 11 12 13 G 1 2 3 4 5 6 7 8 9 10 11 12 13 14 G
F 1 2 3 4 5 6 7 8 9 10 11 12 13 F 1 2 3 4 5 6 7 8 9 10 11 12 13 14 F
E 1 2 3 4 5 6 7 8 9 10 11 12 13 E 1 2 3 4 5 6 7 8 9 10 11 12 13 14 E
D 1 2 3 4 5 6 7 8 9 10 11 12 13 D 1 2 3 4 5 6 7 8 9 10 11 12 13 14 D
C 1 2 3 4 5 6 7 8 9 10 11 12 13 C 1 2 3 4 5 6 7 8 9 10 11 12 13 14 C
B 1 2 3 4 5 6 7 8 9 10 11 12 13 B 1 2 3 4 5 6 7 8 9 10 11 12 13 14 B
A 3 4 5 6 7 8 9 10 11 A 3 4 5 6 7 8 9 10 11 12 13 14 A

FRONT OF ROOM – 2079 Natural History Building
1. (10 points) What is the largest possible area for a rectangle which satisfies all of the following conditions?

- The rectangle’s bottom left corner is at the point (0, 0).
- The rectangle’s top right corner lies on the curve $y = 91e^{-13x}$ for $x > 0$.
- The rectangle’s bottom side lies on the x-axis.

\[ y = 91e^{-13x} \]

\[
\text{area (rectangle)} = \text{width} \cdot \text{height} = x \cdot y = x \cdot 91e^{-13x}
\]

maximize $A = 91xe^{-13x}$ on $(0, \infty)$

\[
A' = 91e^{-13x} + 91x \cdot (-13e^{-13x}) = 91e^{-13x}(1 - 13x)
\]

$A' = 0$ when $1 - 13x = 0 \Rightarrow x = \frac{1}{13}$

values of $A'$:

\[\begin{array}{c|c}
0 & + \\
\frac{1}{13} & - \\
\end{array}\]

at $x = \frac{1}{13}$, we get $\max \text{ area } = A(\frac{1}{13}) = 91(\frac{1}{13})e^{-13(\frac{1}{13})} = \frac{7e^{-1}}{2}$
2. (10 points) A circle is growing in size so that its area increases at a constant rate of 486π \( cm^2/min \). How quickly is the circle’s radius increasing when the circle’s area is 81π \( cm^2 \)?

Given \( \frac{dA}{dt} = 486\pi \text{ cm}^2/\text{min} \)

We want \( \frac{dr}{dt} \) \( A=81\pi \text{ cm}^2 \) \( \Rightarrow \pi r^2 = 81\pi \Rightarrow r = 9 \text{ cm} \)

\( A = \pi r^2 \)

\( \frac{dA}{dt} = \frac{d}{dt}(\pi r^2) \)

\( \frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt} \)

\( \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \)

\( 486\pi = 2\pi (9) \frac{dr}{dt} \)

\( \frac{dr}{dt} = \frac{486\pi}{18\pi} = \frac{486}{18} = \frac{243}{9} = 27 \)

\( \frac{dr}{dt} = 27 \text{ cm/ min} \)
3. (10 points) Find the equation of the line tangent to the given curve at the point \( (x, y) = (1, 0) \).

\[-x^{15}y^3 + 8y + 9 = 9x^{25}\]

\[
\frac{d}{dx}(x^{15}y^3 + 8y + 9) = \frac{d}{dx}(9x^{25})
\]

\[15x^4y^3 + 15x^{15}y^2 \frac{dy}{dx} + 8 \frac{dy}{dx} + 0 = 225x^{24}\]

\[(3x^{15}y^2 + 8) \frac{dy}{dx} = 225x^{24} - 15x^4y^3\]

\[\frac{dy}{dx} = \frac{225x^{24} - 15x^4y^3}{3x^{15}y^2 + 8}\]

\[\frac{dy}{dx}_{(x,y) = (1,0)} = \frac{225(1)^{24} - 15(1)^4(0)^3}{3(1)^{15}(0)^2 + 8} = \frac{225}{8}\]

Tangent line: \( y - 0 = \frac{225}{8}(x - 1) \Rightarrow y = \frac{225}{8}x - \frac{225}{8}\)

4. (10 points) Evaluate the definite integral. Simplify your answer.

\[
\int_{0}^{\sqrt{18}} 40xe^{x^2} \, dx = \left[ 5e^{u^2} \right]_{0}^{418}\]

\[= 5e^{418} - 5e^0\]

\[= 5e^{18(3^4)} - 5\cdot1\]

\[= 5 \cdot 3^4 - 5\]

\[= 5 \cdot 81 - 5\]

\[= 405 - 5\]

\[= 400\]
5. (10 points) Determine the area of the region above the x-axis and below the curve $y = \frac{720}{x^4}$ on the interval $[1, 2]$. Simplify your answer.

\[ A = \int_{1}^{2} \frac{720}{x^4} \, dx = \int_{1}^{2} 720x^{-4} \, dx \]

\[ = 720 \cdot \frac{x^{-3}}{-3} \bigg|_{1}^{2} \]

\[ = -240 \cdot \left( \frac{1}{2^3} - \frac{1}{1^3} \right) \]

\[ = -240 \cdot \left( \frac{2^3 - 1}{2^3} \right) \]

\[ = -240 \cdot \frac{8}{2^3} + 240 \]

\[ = -30 + 240 = 210 \]

6. (10 points) Suppose that $f(x)$ is a polynomial which satisfies the following conditions.

\[ \bullet \frac{\int_{0}^{1} f(x) \, dx = 44}{\int_{1}^{2} f(x) \, dx = 6} \]

Evaluate the following quantities.

(a) $\int_{2}^{0} (3f(x) + 10) \, dx = 3 \int_{2}^{0} f(x) \, dx + \int_{2}^{0} 10 \, dx$

\[ = -3 \int_{2}^{0} f(x) \, dx + 10(0 - 2) \]

\[ = -3(50) + 10(0 - 2) = -170 \]

(b) $\int_{1}^{1} 21x^2 f(x^3 + 1) \, dx$

\[ = \int_{1}^{1} f(u) \, du \]

\[ = \int_{0}^{1} 7f(u) \, du \]

\[ = 7 \cdot 6 = 42 \]
7. (10 points) Evaluate the indefinite integral.

\[
\int 42x^3 (x^4 + 5)^8 \, dx = \int x^{10} (x^4 + 5)^8 \, dx \\
= \int (u - 5) u^8 \, du \\
= \int (u^9 - 5u^8) \, du \\
= \frac{1}{10} u^{10} - \frac{5}{9} u^9 + C \\
= \frac{1}{10} (x^{42} + 5)^{10} - \frac{5}{9} (x^{42} + 5)^9 + C
\]

8. (10 points) Evaluate the indefinite integral.

\[
\int 300xe^{-5x} \, dx = \\
= 300x e^{-5x} - \int -5x e^{-5x} 300 \, dx \\
= -60x e^{-5x} + 60 \int e^{-5x} \, dx \\
= -60x e^{-5x} + 60 \cdot \frac{1}{-5} e^{-5x} + C \\
= -60x e^{-5x} - 12e^{-5x} + C
\]
9. (10 points) Find the average value of the function $f(x) = \frac{6}{x}$ on the interval $[e^2, e^9]$. Simplify your answer.

$$f_{ave} = \frac{1}{e^9 - e^2} \int_{e^2}^{e^9} \frac{6}{x} \, dx$$

$$= \frac{1}{e^9 - e^2} \left. 6 \ln x \right|_{e^2}^{e^9}$$

$$= \frac{1}{e^9 - e^2} (6 \ln (e^9) - 6 \ln (e^2))$$

$$= \frac{1}{e^9 - e^2} (6 \cdot 9 - 6 \cdot 2)$$

$$= \frac{42}{e^9 - e^2}$$

10. (10 points) Let $R$ be the finite region bounded by the graphs of $y = 9 \sqrt{x}$, $y = 15 \sqrt{x}$, $x = 5$ and $x = 17$. Set up, but do not evaluate, a definite integral which is equal to the volume of the solid formed when $R$ is revolved around the $x$-axis.

$$V = \int_{5}^{17} (\text{cross-sectional area at } x) \, dx$$

$$V = \int_{5}^{17} \pi (r_{out}^2 - r_{in}^2) \, dx$$

$$V = \int_{5}^{17} \pi ((15 \sqrt{x})^2 - (9 \sqrt{x})^2) \, dx$$

$$V = \int_{5}^{17} \pi (15\sqrt{x})^2 - \pi (9\sqrt{x})^2 \, dx$$
Students – do not write on this page!

1. (10 points) ______________________

2. (10 points) ______________________

3. (10 points) ______________________

4. (10 points) ______________________

5. (10 points) ______________________

6. (10 points) ______________________

7. (10 points) ______________________

8. (10 points) ______________________

9. (10 points) ______________________

10. (10 points) _____________________

TOTAL (100 points) ________________