1. (4 points) Determine the interval of convergence for the series.

\[
\sum_{n=0}^{\infty} \frac{(2x-5)^n}{n+8}
\]

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(2x-5)^{n+1}}{(n+9)} \cdot \frac{(n+8)}{(2x-5)^n} \right|
\]

\[
= \lim_{n \to \infty} \left| \frac{(2x-5)}{n+9} \right|
\]

\[
= |2x-5|
\]

**Series converges absolutely when** \(|2x-5|<1\),

\[-1 < 2x-5 < 1\]

\[4 < 2x < 6\]

\[2 < x < 3\]

**Series diverges when** \(x<2\) or \(x>3\).

\[x=2\] \(\sum (2x-5)^n = \sum (-1)^n \) which converges by alternating series test \((\text{tail end of harmonic series})\)

\[x=3\] \(\sum (2x-5)^n = \sum \frac{1}{n+8} \) which diverges
2. (3 points) Determine whether the series converges or diverges.

\[ 1 + \frac{1}{3!} + \frac{1}{6!} + \frac{1}{9!} + \frac{1}{12!} + \cdots = \sum_{k=0}^{\infty} \frac{1}{(3k)!} \]

\[ \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{1/(3(n+1)!)}{1/(3n)!} = \lim_{n \to \infty} \frac{(3n)!}{(3n+3)!} \]

\[ = \lim_{n \to \infty} \frac{(3n)!}{(3n+3)(3n+2)(3n+1)(3n)!} \]

\[ = \lim_{n \to \infty} \frac{1}{(3n+3)(3n+2)(3n+1)} = 0 < 1 \]

By the Ratio Test, the series converges absolutely.

3. (3 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = -1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \cdots \]

Since \( \lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0 \) and \( \left\{ \frac{1}{\sqrt{n}} \right\} \) is decreasing, the series converges by the alternating series test.

However, \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]

is a p-series with \( p = \frac{1}{2} \), and since it diverges, \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \]

converges conditionally.