• No calculators are allowed.
• Show sufficient work to justify each answer.
• You have 15 minutes for this quiz.

1. (1 point) Suppose that \( |f''(x)| \leq K \) for \( a \leq x \leq b \). If \( E_T \) is the error involved in using the Trapezoidal Rule, then which one of the following is the error bound given in the book?

(a) \( |E_T| \leq \frac{K(b-a)^2}{12n^3} \)  
(b) \( |E_T| > \frac{K(b-a)^2}{12n^3} \)

(c) \( |E_T| \leq \frac{K(b-a)^5}{180n^4} \)  
(d) \( |E_T| > \frac{K(b-a)^5}{180n^4} \)

(e) \( |E_T| \leq \frac{K(b-a)^3}{12n^2} \)  
(f) \( |E_T| > \frac{K(b-a)^3}{12n^2} \)

(g) \( |E_T| \leq \frac{K(b-a)^3}{24n^2} \)  
(h) \( |E_T| > \frac{K(b-a)^3}{24n^2} \)

(i) \( |E_T| \leq \frac{K(b-a)}{n^4} \)  
(j) \( |E_T| > \frac{K(b-a)}{n^4} \)

2. (1 point) Suppose that \( |f^{(i)}(x)| \leq K \) for \( a \leq x \leq b \). If \( E_S \) is the error involved in using Simpson’s Rule, then which one of the following is the error bound given in the book?

(a) \( |E_S| \leq \frac{K(b-a)^2}{12n^3} \)  
(b) \( |E_S| > \frac{K(b-a)^2}{12n^3} \)

(c) \( |E_S| \leq \frac{K(b-a)^5}{180n^4} \)  
(d) \( |E_S| > \frac{K(b-a)^5}{180n^4} \)

(e) \( |E_S| \leq \frac{K(b-a)^3}{12n^2} \)  
(f) \( |E_S| > \frac{K(b-a)^3}{12n^2} \)

(g) \( |E_S| \leq \frac{K(b-a)^3}{24n^2} \)  
(h) \( |E_S| > \frac{K(b-a)^3}{24n^2} \)

(i) \( |E_S| \leq \frac{K(b-a)}{n^4} \)  
(j) \( |E_S| > \frac{K(b-a)}{n^4} \)
3. (4 points) If the Midpoint Rule is used to approximate $\int_2^4 \frac{1}{x^3} \, dx$, how large must $n$ be so that the approximation is accurate to within $1/80000 = 0.0000125$? Simplify your answer.

\[
\begin{align*}
    f(x) &= x^{-3} = x^{-3} \\
    f'(x) &= -3x^{-4} \\
    f''(x) &= 12x^{-5} = \frac{12}{x^5} \\
    \text{For } 2 \leq x \leq 4, \\
    |f''(x)| &= \frac{12}{2^5} = \frac{12}{32} = \frac{3}{8} \\
    \text{let } k &= \frac{3}{8} = \frac{3}{8}
\end{align*}
\]

\[
\begin{align*}
    1E_m &\leq \frac{K(b-a)^3}{24n^2} \\
    1E_m &\leq \frac{3}{8} (4-2)^3 \\
    1E_m &\leq \frac{1}{8n^2} \\
    \frac{1}{8n^2} &\leq \frac{1}{80000} \\
    \frac{1}{n^2} &\leq \frac{1}{10000} \\
    n^2 &\geq 10000 \\
    n &\geq 100
\end{align*}
\]

4. (2 points each) A curve is defined by the function $y = e^{2x}$ for $0 \leq x \leq 2$. Set up, but do not evaluate, an integral for the area of the surface obtained by rotating this curve about

(a) the $x$-axis

\[
S = \int_0^2 2\pi y \sqrt{1 + (2e^{2x})^2} \, dx
\]

(b) the $y$-axis

\[
S = \int_0^2 2\pi x \sqrt{1 + (2e^{2x})^2} \, dx
\]

or

\[
S = \int_0^2 e^x 2\pi y \sqrt{1 + (\frac{1}{2y})^2} \, dy
\]