Name ____________________________

- No calculators are allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 points each) Determine whether the integrals converge or diverge. For a convergent integral you must determine its value. Use proper notation for each step in your work.

   \[
   (a) \int_2^6 \frac{dx}{(x-2)^{3/2}} = \lim_{a \to 2^+} \int_a^6 \frac{dx}{(x-2)^{3/2}} \\
   = \lim_{a \to 2^+} \int_a^4 \frac{du}{u^{3/2}} \\
   = \lim_{a \to 2^+} \left[ -2u^{-1/2} \right]_a^4 \\
   = \lim_{a \to 2^+} \left[ \frac{-2}{\sqrt{4}} - \frac{-2}{\sqrt{a-2}} \right] \\
   = \lim_{a \to 2^+} \left[ -1 + \frac{2}{\sqrt{a-2}} \right] \\
   = \infty \quad \text{(diverges)}
   \]
2. (4 points) The region between the x-axis and the curve \( y = e^{-x} \) on the interval \([0, \infty)\) is revolved around the x-axis. Compute the resulting volume.

\[
V = \int_0^\infty \pi (e^{-x})^2 \, dx = \pi \int_0^\infty e^{-2x} \, dx = \lim_{b \to \infty} \int_0^b \pi e^{-2x} \, dx = \lim_{b \to \infty} \left[ -\frac{\pi}{2} e^{-2x} \right]_0^b = \lim_{b \to \infty} \left[ -\frac{\pi}{2} e^{-2b} + \frac{\pi}{2} e^0 \right] = \frac{\pi}{2} \]