1. For each series, prove whether it converges or diverges.

(a) \[ \sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k} = \frac{\cos(1\pi) + \cos(2\pi) + \cos(3\pi) + \cdots}{1 + \frac{1}{2} + \frac{1}{3} + \cdots} = -1 + \frac{1}{2} - \frac{1}{3} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \]

This is an alternating series where \(\frac{1}{k}\) is decreasing with \(\lim_{k\to\infty} \frac{1}{k} = 0\).

The series converges by the alternating series test.

(b) \[ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \ln k \]

\[ \lim_{k\to\infty} \frac{(-1)^{k+1} k}{\ln k} = \lim_{k\to\infty} \frac{k - \infty}{\ln k} = \infty \]

since the terms do not approach 0, the series diverges by the divergence test.

(Or use L'Hôpital's rule on \(\frac{k}{\ln k}\).)
2. The following alternating series converges to a sum $S$.

$$S = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}} + \cdots$$

(a) If we approximate the sum of the infinite series with

$$S \approx \sum_{k=1}^{3} (-1)^{k+1} \frac{1}{\sqrt{k}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$$

then how can the error in our approximation be bounded? (fill in the blank below)

$$|\text{error}| \leq \frac{1}{\sqrt{4}} = 0.5$$

(b) How many of the beginning terms of the infinite series should you add together to get an estimate for its sum $S$ that is guaranteed to be within 0.01 of the correct sum?

$$S = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots + (-1)^{n+1} \frac{1}{\sqrt{n}}$$

$$|\text{error}| \leq \frac{1}{\sqrt{n+1}} \leq 0.01$$

$$\frac{1}{\sqrt{n+1}} \leq \frac{1}{100}$$

$$\sqrt{n+1} \geq 100$$

$$n+1 \geq 10000$$

$$n \geq 9999$$

adding the first 9999 terms is sufficient to guarantee $|\text{error}| = 0.01$