Math 220 (section AD?) Quiz 9 Spring 2019

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{x^5+x^3+2}{x^2+1}\,dx = \int \left(x^3 + \frac{2}{x^2+1}\right)dx = \int x^3 dx + 2\int \frac{1}{x^2+1}\,dx$

$= \frac{1}{4}x^4 + 2\arctan(x) + C$

Polynomial long division

$$\begin{array}{r} x^3 \\ x^2+1 \overline{\big)\, x^5+x^3+2} \\ -(x^5+x^3) \\ \hline 2 \end{array}$$

$$\frac{x^5+x^3+2}{x^2+1} = x^3 + \frac{2}{x^2+1}$$

(b) $\int \tan(x)\csc(2x)\,dx = \int \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\sin(2x)}\,dx$

$= \int \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{2\sin(x)\cos(x)}\,dx$

$= \frac{1}{2}\int \frac{1}{\cos^2(x)}\,dx$

$= \frac{1}{2}\int \sec^2(x)\,dx$

$= \frac{1}{2}\tan(x) + C$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_1^3 2x^{-3}\left(x^2+18\right)\,dx = \int_1^3 \left(2x^{-1}+36x^{-3}\right)dx$$

$$= 2\int_1^3 \frac{1}{x}\,dx + 36\int_1^3 x^{-3}\,dx$$

$$= 2\left[\ln|x|\right]_1^3 + 36\left[\frac{x^{-2}}{-2}\right]_1^3$$

$$= 2\left[\ln(3)-\ln(1)\right] + 36\left[\frac{3^{-2}}{-2}-\frac{1^{-2}}{-2}\right]$$

$$= 2\left[\ln(3)-0\right] + 36\left[-\frac{1}{18}-\left(\frac{-1}{2}\right)\right]$$

$$= 2\ln(3) \quad -2+18$$

$$= 2\ln(3)+16$$

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3. (2 points) Suppose $g(x) = \int_{x^6}^{4} e^{\sqrt[3]{t}}\,dt$. Find $g'(x)$.

$$g(x) = -\int_4^{x^6} e^{\sqrt[3]{t}}\,dt$$

$$g'(x) = -e^{\sqrt[3]{x^6}} \cdot \frac{d}{dx}(x^6)$$

(by the Fundamental theorem of Calculus (part 1) and the chain rule)

$$g'(x) = -e^{\sqrt[3]{x^6}} \cdot 6x^5$$

$$g'(x) = -6x^5 e^{x^2}$$

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4. (2 points) At time t minutes, the altitude of a drone is changing at a rate of $10\sin(t) + 20\cos(t)$ feet per minute. If the altitude of the drone is 200 feet at time $t = \frac{\pi}{2}$ minutes, then what is its altitude at time $t = 2\pi$ minutes? Simplify your answer.

$$\begin{pmatrix}\text{altitude}\\ \text{at } t=2\pi\end{pmatrix} = \begin{pmatrix}\text{altitude}\\ \text{at } t=\frac{\pi}{2}\end{pmatrix} + \begin{pmatrix}\text{change in altitude}\\ \text{between } t=\frac{\pi}{2} \text{ and } t=2\pi\end{pmatrix}$$

$$= 200 + \int_{\frac{\pi}{2}}^{2\pi} \begin{pmatrix}\text{rate of change}\\ \text{in altitude}\end{pmatrix} dt \quad \begin{pmatrix}\text{by net}\\ \text{change}\\ \text{theorem}\end{pmatrix}$$

$$= 200 + \int_{\pi/2}^{2\pi} (10\sin(t) + 20\cos(t))\,dt$$

$$= 200 + \Big[-10\cos(t) + 20\sin(t)\Big]_{\pi/2}^{2\pi}$$

$$= 200 + \left[\left(-10\cos(2\pi) + 20\sin(2\pi)\right) - \left(-10\cos(\tfrac{\pi}{2}) + 20\sin(\tfrac{\pi}{2})\right)\right]$$

$$= 200 + \left[(-10(1) + 20(0)) - (-10(0) + 20(1))\right]$$

$$= 200 + [-10 - 20]$$

$$= 170 \text{ feet}$$

alternate approach

$$h'(t) = 10\sin(t) + 20\cos(t)$$

$$\Rightarrow h(t) = -10\cos(t) + 20\sin(t) + C$$

$$h(\tfrac{\pi}{2}) = 200 \Rightarrow C = 180$$

$$h(t) = -10\cos(t) + 20\sin(t) + 180$$

$$h(2\pi) = 170 \text{ feet}$$