Math 220 **Quiz 7 (take-home)** **Spring 2019**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Ran Ji
- ▷ **AD2**, TR 1:00-2:50, Cassie Christenson
- ▷ **AD3**, TR 11:00-12:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Eion Blanchard
- ▷ **ADB**, TR 9:00-9:50, Eion Blanchard
- ▷ **ADC**, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- ▷ **ADD**, TR 11:00-11:50, Stathis Chrontsios
- ▷ **ADE**, TR 12:00-12:50, Kesav Krishnan
- ▷ **ADF**, TR 1:00-1:50, Souktik Roy
- ▷ **ADG**, TR 2:00-2:50, Gidon Orelowitz
- ▷ **ADH**, TR 3:00-3:50, Mina Nahvi
- ▷ **ADJ**, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- ▷ **ADK**, TR 10:00-10:50, Souktik Roy
- ▷ **ADL**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADM**, TR 12:00-12:50, Vincent Villalobos
- ▷ **ADN**, TR 1:00-1:50, Kesav Krishnan
- ▷ **ADO**, TR 2:00-2:50, Stathis Chrontsios
- ▷ **ADQ**, TR 4:00-4:50, Mina Nahvi
- ▷ **ADR**, TR 10:00-10:50, Vincent Villalobos

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write their solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your lecture period on Friday, March 8th.**
- **TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (9-9:50am, 10-10:50am).**

1. (2 points) Complete the sentences about the given function. You must fully justify each answer.

$$f(x) = 6xe^{-3x} + 2e^{-3x}$$

(a) The function f is decreasing on the interval $(0, \infty)$

(b) The function f is increasing on the interval $(-\infty, 0)$

(c) The function f is concave down on the interval $(-\infty, \frac{1}{3})$

(d) The function f is concave up on the interval $(\frac{1}{3}, \infty)$

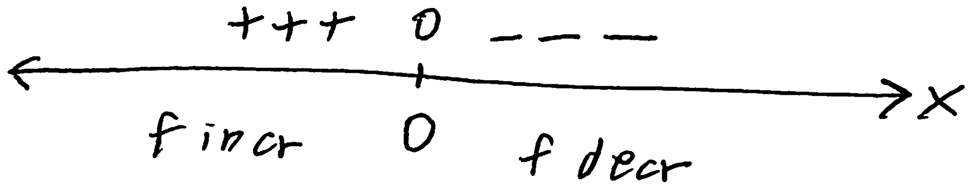
$$f'(x) = 6e^{-3x} + 6xe^{-3x}\cdot(-3) + 2e^{-3x}\cdot(-3)$$

$$f'(x) = -18x e^{-3x}$$

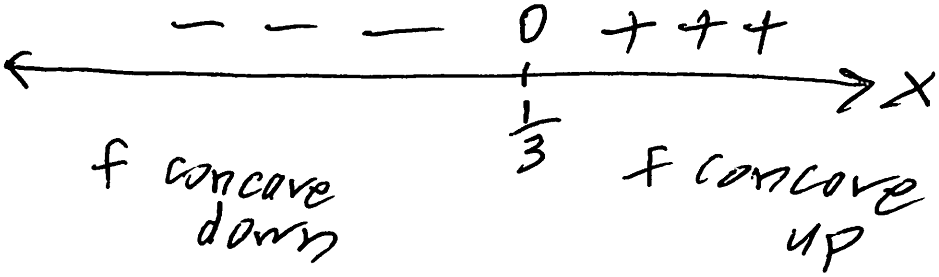
$$f''(x) = -18e^{-3x} + (-18x)e^{-3x}\cdot(-3)$$

$$= 18e^{-3x}(-1+3x)$$

values of $f'(x) = -18x e^{-3x}$



values of $f''(x) = 18e^{-3x}(-1+3x)$



2. (4 points) Evaluate the following limit.

$$\lim_{x\to 1} \left(\ln\left(x^6\right)+1\right)^{1/\ln\left(x^3\right)}$$

(indeterminate form; $\underset{x\to 1^+}{1^{\infty}}$ or $\underset{x\to 1^-}{1^{-\infty}}$)

$$\lim_{x\to 1} \left(\ln(x^6)+1\right)^{1/\ln(x^3)} = \lim_{x\to 1} e^{\ln\left(\left(\ln(x^6)+1\right)^{1/\ln(x^3)}\right)}$$

using $u=e^{\ln(u)}$

$$\equiv e^{\lim_{x\to 1} \ln\left(\left(\ln(x^6)+1\right)^{1/\ln(x^3)}\right)}$$

$$= e^{\lim_{x\to 1} \frac{1}{\ln(x^3)} \cdot \ln\left(\ln(x^6)+1\right)}$$

$$= e^{\lim_{x\to 1} \frac{\ln\left(\ln(x^6)+1\right)}{\ln(x^3)}} \quad \begin{matrix}\to 0 \\ \to 0\end{matrix} \quad \left(\begin{matrix}\text{indeterminate} \\ \text{form } \frac{0}{0}\end{matrix}\right)$$

$$\overset{H}{=} e^{\lim_{x\to 1} \frac{\frac{d}{dx}\left(\ln\left(\ln(x^6)+1\right)\right)}{\frac{d}{dx}\left(\ln(x^3)\right)}}$$

by l'Hospital's Rule

$$= e^{\lim_{x\to 1} \frac{\frac{1}{\ln(x^6)+1} \cdot \frac{1}{x^6} \cdot 6x^5}{\frac{1}{x^3} \cdot 3x^2}}$$

$$= e^{\lim_{x\to 1} \frac{2}{\ln(x^6)+1}}$$

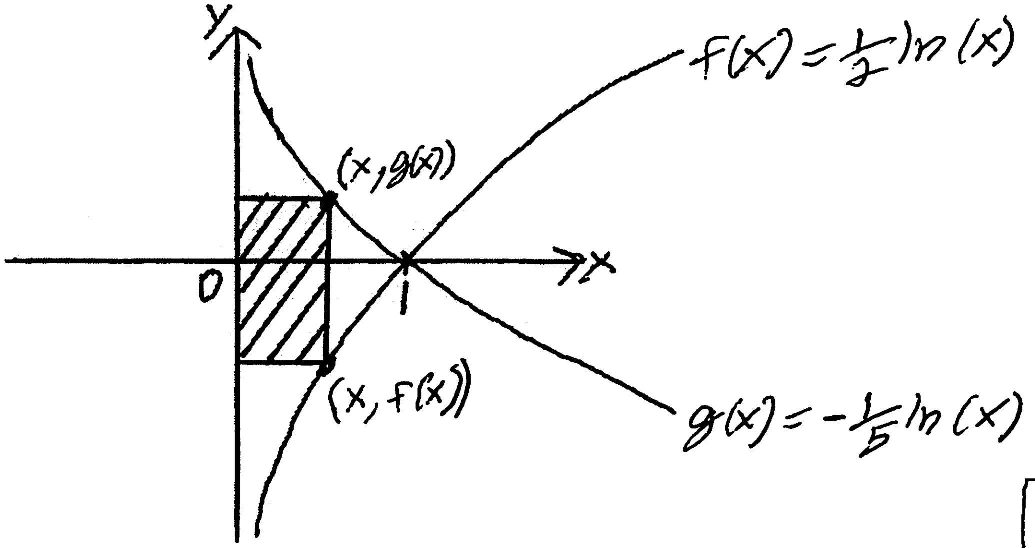
$$= e^{2}$$

3. **(4 points)** Suppose $f(x) = \frac{1}{2}\ln(x)$ and $g(x) = -\frac{1}{5}\ln(x)$.

For each x in the interval $(0, 1)$, consider the rectangle formed in the following manner.

- Its right side is the line segment connecting the points $(x, f(x))$ and $(x, g(x))$.
- Its left side lies along the y-axis.

Which value of x in the interval $(0, 1)$ results in the rectangle of largest area?



Area = width · height

$A = x\cdot(g(x) - f(x))$

$A = x(-\frac{1}{5}\ln(x) - \frac{1}{2}\ln(x))$

$A = -\frac{7}{10}x\ln(x)$

maximize $A = -\frac{7}{10}x\ln(x)$ on $(0,1)$

$A' = -\frac{7}{10}\ln(x) + (-\frac{7}{10}x\cdot\frac{1}{x})$

$A' = -\frac{7}{10}(\ln(x)+1)$

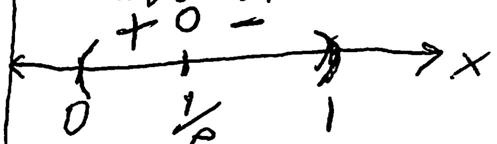
$A' = 0 \Rightarrow \ln(x)+1 = 0$

$\ln(x) = -1$

$x = e^{-1}$

$= 1/e$

values of A'



A is increasing on $(0, \frac{1}{e})$

A is decreasing on $(\frac{1}{e}, 1)$

The maximum area occurs when $x = \frac{1}{e}$

note; without a calculator, some students evaluated A' at $\frac{1}{e^2}$ and $\frac{1}{\sqrt{e}}$ to determine the sign to the left and right of 1/e