Math 220 Quiz 7 (take-home) Spring 2019

Name Solutions

(circle your TA discussion section)

• AD1, TR 9:00-10:50, Ran Ji
• AD2, TR 1:00-2:50, Cassie Christenson
• AD3, TR 11:00-12:50, Dana Neidinger
• ADA, TR 8:00-8:50, Eion Blanchard
• ADB, TR 9:00-9:50, Eion Blanchard
• ADC, TR 10:00-10:50, Yuxuan "Yuki" Zhang
• ADD, TR 11:00-11:50, Stathis Chrontsios
• ADE, TR 12:00-12:50, Kesav Krishnan
• ADF, TR 1:00-1:50, Souktik Roy
• ADG, TR 2:00-2:50, Gidon Orelowitz
• ADH, TR 3:00-3:50, Mina Nahvi
• ADJ, TR 9:00-9:50, Yuxuan "Yuki" Zhang
• ADK, TR 10:00-10:50, Souktik Roy
• ADL, TR 11:00-11:50, Gidon Orelowitz
• ADM, TR 12:00-12:50, Vincent Villalobos
• ADN, TR 1:00-1:50, Kesav Krishnan
• ADO, TR 2:00-2:50, Stathis Chrontsios
• ADQ, TR 4:00-4:50, Mina Nahvi
• ADR, TR 10:00-10:50, Vincent Villalobos

• You may lose points if you do not circle your correct discussion section.

• You may work with other MATH 220 students. However each student should write their solutions separately and independently – nobody should copy someone else’s work.

• You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.

• You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.

• There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.

• Be sure that the pages are nicely stapled – do not just fold the corners.

• The quiz is due at the beginning of your lecture period on Friday, March 8th.

• TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (9-9:50am, 10-10:50am).
1. (2 points) Complete the sentences about the given function. You must fully justify each answer.

\[ f(x) = 6xe^{-3x} + 2e^{-3x} \]

(a) The function \( f \) is decreasing on the interval \((0, \infty)\)

(b) The function \( f \) is increasing on the interval \((-\infty, 0)\)

(c) The function \( f \) is concave down on the interval \((-\infty, \frac{1}{3})\)

(d) The function \( f \) is concave up on the interval \((\frac{1}{3}, \infty)\)

\[ f'(x) = 6e^{-3x} + 6xe^{-3x}(-3) + 2e^{-3x}(-3) \]
\[ f'(x) = -18xe^{-3x} \]

\[ f''(x) = -18e^{-3x} + (-18x)e^{-3x}, (-3) \]
\[ f''(x) = 18e^{-3x}(-1 + 3x) \]

Values of \( f'(x) = -18xe^{-3x} \)

\[ f \text{ incr} \quad 0 \quad f \text{ decr} \]

Values of \( f''(x) = 18e^{-3x}(-1 + 3x) \)

\[ f \text{ concave down} \quad \frac{1}{3} \quad f \text{ concave up} \]
2. (4 points) Evaluate the following limit.

\[
\lim_{x \to 1} \frac{(\ln(x^6) + 1)^{1/\ln(x^3)}}{\ln(x^3)}
\]

(indeterminate form: \( \frac{\infty}{\infty} \) or \( \frac{1}{-\infty} \) or \( \frac{1}{1^0} \))

Using \( u = e^{\ln(u)} \)

\[
\lim_{x \to 1} \frac{\ln(\ln(x^6) + 1)^{1/\ln(x^3)}}{\ln(x^3)}
\]

\[
\lim_{x \to 1} \frac{\ln(\ln(x^6) + 1)}{\ln(x^3)}
\]

\[
\lim_{x \to 1} \frac{\ln(\ln(x^6) + 1)}{\ln(x^3)} \to 0
\]

(indeterminate form \( \frac{0}{0} \))

by l'Hopital's Rule

\[
\lim_{x \to 1} \frac{\frac{d}{dx} (\ln(\ln(x^6) + 1))}{\frac{d}{dx} (\ln(x^3))}
\]

\[
\lim_{x \to 1} \frac{\frac{1}{\ln(x^6) + 1} \cdot 6x^5}{\frac{1}{x^3} \cdot 3x^2}
\]

\[
\lim_{x \to 1} \frac{2}{\ln(x^6) + 1}
\]

\[
e^2
\]
3. (4 points) Suppose \( f(x) = \frac{1}{2} \ln(x) \) and \( g(x) = -\frac{1}{2} \ln(x) \).

For each \( x \) in the interval \( (0, 1) \), consider the rectangle formed in the following manner.

- Its right side is the line segment connecting the points \((x, f(x))\) and \((x, g(x))\).
- Its left side lies along the \( y \)-axis.

Which value of \( x \) in the interval \( (0, 1) \) results in the rectangle of largest area?

\[
\text{Area} = \text{width} \cdot \text{height} \\
A = x \cdot (g(x) - f(x)) \\
A = x \left(-\frac{1}{2} \ln(x) - \frac{1}{2} \ln(x)\right) \\
A = -\frac{7}{10} x \ln(x) \\
\text{maximize } A = -\frac{7}{10} x \ln(x) \text{ on } (0, 1) \\
A' = -\frac{7}{10} \ln(x) + \left(-\frac{7}{10} x \cdot \frac{1}{x}\right) \\
A' = -\frac{7}{10} (\ln(x) + 1) \\
A' = 0 \Rightarrow \ln(x) + 1 = 0 \\
\ln(x) = -1 \\
x = e^{-1} \\
x = \frac{1}{e}
\]

Note: Without a calculator, some students evaluated \( A' \) at \( \frac{1}{e^2} \) and \( \frac{1}{5e} \) to determine the sign to the left and right of \( \frac{1}{e} \).