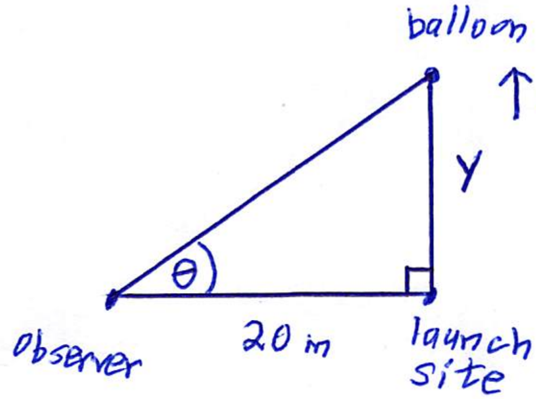
Math 220 (section AD?) **Quiz 6** **Spring 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (4 points) There is a launch site for a hot-air balloon on the ground 20 meters away from an observer. The balloon rises vertically at a constant rate of 2 meters per second. How quickly is the angle of elevation of the balloon increasing 5 seconds after its launch?



Given: $\frac{dy}{dt} = 2 \text{ m/s}$

Want: $\left.\frac{d\theta}{dt}\right|_{t=5s}$

$\tan(\theta) = \frac{y}{20} = \frac{1}{20}y$

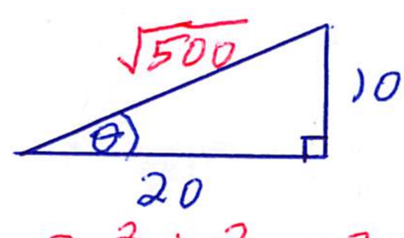
$\frac{d}{dt}(\tan(\theta)) = \frac{d}{dt}\left(\frac{1}{20}y\right)$

$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{20}\frac{dy}{dt}$

note: at $t=5s$,

$y = \left(2\frac{m}{s}\right)(5s) = 10 \text{ m}$

so we have



$20^2 + 10^2 = c^2$

$c = \sqrt{500}$

$\left(\frac{\sqrt{500}}{20}\right)^2 \frac{d\theta}{dt} = \frac{1}{20}(2)$

$\frac{500}{400}\frac{d\theta}{dt} = \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{4}{5} = \frac{2}{25} \text{ rad/s}$

The angle is increasing at $\frac{2}{25}$ rad/s

2. (3 points) At each point on the curve $y = f(x)$, the slope of the curve is equal to its y-coordinate multiplied by $1/4$. If its graph goes through the point $(\ln(81), 36)$, then find a formula for $f(x)$. Simplify your answer.

$$\frac{dy}{dx} = \frac{1}{4}y \Rightarrow y = Ce^{\frac{1}{4}x}$$

$$(x,y) = (\ln(81), 36) \Rightarrow 36 = Ce^{\frac{1}{4}\ln(81)}$$

$$C = \frac{36}{e^{\frac{1}{4}\ln(81)}}$$

$$= \frac{36}{e^{\frac{1}{4}\ln(3^4)}}$$

$$= \frac{36}{e^{\ln(3)}}$$

$$= \frac{36}{3}$$

$$= 12$$

$$y = 12e^{\frac{1}{4}x}$$

3. (3 points) A bullet is shot upward from the surface of a planet so that its height in meters until coming to rest is given by the equation $s(t) = 195t - 6.5t^2$ where t is measured in seconds. At what time does the bullet reach its maximum height?

(height) $s(t) = 195t - 6.5t^2$

(velocity) $s'(t) = 195 - 13t$

$s'(t) = 13(15 - t)$

values of $s'(t)$

$+ + +$ 0 $- - -$

$\longrightarrow x$

15

$s(t)$ is increasing

$s(t)$ is decreasing

The bullet reaches its maximum height at $t = 15$ seconds