Math 220 (section AD?) **Quiz 5** **Spring 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Compute $w'(t)$ for the given function.

$$w(t) = \sqrt[3]{\sin(t^5 + 4t)} = (\sin(t^5+4t))^{1/3}$$

$$w'(t) = \frac{1}{3}(\sin(t^5+4t))^{-2/3} \cdot \frac{d}{dt}(\sin(t^5+4t))$$

$$w'(t) = \frac{1}{3}(\sin(t^5+4t))^{-2/3} \cdot \cos(t^5+4t) \cdot \frac{d}{dt}(t^5+4t)$$

$$w'(t) = \frac{1}{3}(\sin(t^5+4t))^{-2/3} \cdot \cos(t^5+4t) \cdot (5t^4+4)$$

2. (3 points) Compute the second derivative $g''(x)$ for the given function.

$g(x) = \arctan\left(3e^{2x}\right)$

$$g'(x) = \frac{1}{(3e^{2x})^2+1} \cdot \frac{d}{dx}(3e^{2x})$$

$$g'(x) = \frac{1}{(3e^{2x})^2+1} \cdot 3e^{2x} \cdot \frac{d}{dx}(2x)$$

$$g'(x) = \frac{1}{(3e^{2x})^2+1} \cdot 3e^{2x} \cdot 2$$

$$g'(x) = \frac{6e^{2x}}{9e^{4x}+1}$$

$$g''(x) = \frac{\frac{d}{dx}(6e^{2x}) \cdot (9e^{4x}+1) - 6e^{2x} \cdot \frac{d}{dx}(9e^{4x}+1)}{(9e^{4x}+1)^2}$$

$$g''(x) = \frac{6e^{2x} \cdot 2 \cdot (9e^{4x}+1) - 6e^{2x} \cdot 9e^{4x} \cdot 4}{(9e^{4x}+1)^2}$$

3. (3 points) Find the equation of the line tangent to the given curve at the point $(-1, 2)$.

$2xy^3 = 5x^3y - 6$

$$\frac{d}{dx}(2xy^3) = \frac{d}{dx}(5x^3y - 6)$$

$$\frac{d}{dx}(2x)\cdot y^3 + 2x\cdot\frac{d}{dx}(y^3) = \frac{d}{dx}(5x^3)\cdot y + 5x^3\cdot\frac{d}{dx}(y) - 0$$

$$2\cdot y^3 + 2x\cdot 3y^2\frac{dy}{dx} = 15x^2\cdot y + 5x^3\cdot\frac{dy}{dx}$$

$$2y^3 + 6xy^2\frac{dy}{dx} = 15x^2y + 5x^3\frac{dy}{dx}$$

$$6xy^2\frac{dy}{dx} - 5x^3\frac{dy}{dx} = 15x^2y - 2y^3$$

$$\frac{dy}{dx}(6xy^2 - 5x^3) = 15x^2y - 2y^3$$

$$\frac{dy}{dx} = \frac{15x^2y - 2y^3}{6xy^2 - 5x^3}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(-1,2)} = \frac{15(-1)^2(2) - 2(2)^3}{6(-1)(2)^2 - 5(-1)^3} = \frac{30-16}{-24+5} = -\frac{14}{19}$$

Point: $(-1,2)$

slope: $-\frac{14}{19}$

line: $y - 2 = \frac{-14}{19}(x - (-1))$

$$y = -\frac{14}{19}(x+1) + 2$$

$$y = \frac{-14}{19}x + \frac{24}{19}$$

4. (2 points) Compute $\frac{dy}{dx}$ for the given function. Write your answer completely in terms of x.

$$y = \left(e^{9x} + 5\right)^{x^2}$$

$$\ln(y) = \ln\left(\left(e^{9x}+5\right)^{x^2}\right)$$

$$\ln(y) = x^2 \cdot \ln(e^{9x}+5)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(x^2 \cdot \ln(e^{9x}+5)\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x^2) \cdot \ln(e^{9x}+5) + x^2 \cdot \frac{d}{dx}\left(\ln(e^{9x}+5)\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \cdot \ln(e^{9x}+5) + x^2 \cdot \frac{1}{e^{9x}+5} \cdot \frac{d}{dx}(e^{9x}+5)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \cdot \ln(e^{9x}+5) + x^2 \cdot \frac{1}{e^{9x}+5} \cdot e^{9x} \cdot 9$$

$$\frac{1}{y} \frac{dy}{dx} = 2x \ln(e^{9x}+5) + \frac{9x^2 e^{9x}}{e^{9x}+5}$$

$$\frac{dy}{dx} = y\left(2x \ln(e^{9x}+5) + \frac{9x^2 e^{9x}}{e^{9x}+5}\right)$$

$$\frac{dy}{dx} = (e^{9x}+5)^{x^2}\left(2x \ln(e^{9x}+5) + \frac{9x^2 e^{9x}}{e^{9x}+5}\right)$$