Math 220 (section AD?) **Quiz 4** **Spring 2019**

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Let $f(x) = x^3 + 42$ and $g(x) = 6x^2 - 12x$. At one particular x-value, the line tangent to $f(x)$ and the line tangent to $g(x)$ are parallel. Determine that x-value.

Parallel lines have the same slope, since the derivative gives a formula for the slopes, we set $f'(x) = g'(x)$

$f'(x) = 3x^2$, $g'(x) = 12x - 12$

$f'(x) = g'(x) \Rightarrow 3x^2 = 12x - 12$

$\Rightarrow 3x^2 - 12x + 12 = 0$

$\Rightarrow 3(x^2 - 4x + 4) = 0$

$\Rightarrow 3(x-2)^2 = 0 \Rightarrow x = 2$

2. (2 points) What is the slope of the following curve at $x = \pi/3$? Simplify your answer.

$$y = 5\tan(x) + \underbrace{\cos(2\arctan(7/24))}_{\text{constant}}$$

$y' = 5\sec^2(x) + 0$

$y'(\pi/3) = 5\sec^2(\pi/3)$

$= \dfrac{5}{\cos^2(\pi/3)}$

$= \dfrac{5}{(1/2)^2}$

$= 20$

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $p = \left(\frac{\sqrt[4]{x}}{x\sqrt{x^3}}\right)^8$ (simplify your answer)

$$p = \left(\frac{x^{1/4}}{x^{5/2}}\right)^8$$

$$p = \frac{(x^{1/4})^8}{(x^{5/2})^8}$$

$$p = \frac{x^2}{x^{20}}$$

$$p = \frac{1}{x^{18}}$$

$$p = x^{-18}$$

$$\frac{dp}{dx} = -18x^{-19}$$

or $\frac{-18}{x^{19}}$

(b) $\theta = \frac{4t}{t^6 + 3e^t}$

$$\frac{d\theta}{dt} = \frac{\frac{d}{dt}(4t)\cdot(t^6+3e^t) - 4t\cdot\frac{d}{dt}(t^6+3e^t)}{(t^6+3e^t)^2}$$

$$\frac{d\theta}{dt} = \frac{4(t^6+3e^t) - 4t(6t^5+3e^t)}{(t^6+3e^t)^2}$$

(c) $Z = 5\csc(w) + 3\sqrt{w}\sin(w)$

$$\frac{dZ}{dw} = 5(-\csc(w)\cot(w)) + \frac{d}{dw}(3\sqrt{w})\cdot\sin(w) + 3\sqrt{w}\cdot\frac{d}{dw}(\sin(w))$$

$$\frac{dZ}{dw} = -5\csc(w)\cot(w) + \frac{3}{2}w^{-1/2}\sin(w) + 3\sqrt{w}\cos(w)$$