Math 220 (section AD?) Quiz 3 Spring 2019

Name Solutions

- 20 minutes
- No calculators
- Show sufficient work

We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

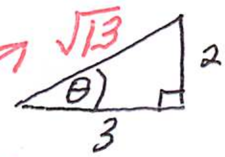
1. (2 points) Evaluate $\csc(2\arctan(2/3))$.

Let $\theta = \arctan(2/3)$

Then $\tan(\theta) = \frac{2}{3}$ $\left(\frac{\text{opp}}{\text{adj}}\right)$

with $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{2}{3} > 0 \Rightarrow \theta \in (0, \pi/2)$



From Pythagorean theorem

$$\csc\left(2\arctan\left(\tfrac{2}{3}\right)\right) = \csc(2\theta)$$

$$= \frac{1}{\sin(2\theta)}$$

$$= \frac{1}{2\sin(\theta)\cos(\theta)}$$

$$= \frac{1}{2\left(\frac{2}{\sqrt{13}}\right)\left(\frac{3}{\sqrt{13}}\right)}$$

$$= \frac{1}{12/13} = \frac{13}{12}$$

2. (2 points) Write an equation for each vertical asymptote on the graph of the given function. Use limits to justify your answer.

$$f(x) = \frac{10x^2 - 90}{x^2 - 8x + 15} = \frac{10(x^2-9)}{(x-5)(x-3)} = \frac{10(x+3)(x-3)}{(x-5)(x-3)} = \frac{10(x+3)}{x-5}$$

$f(x)$ is undefined at $x=3$ and $x=5$, so we will check for vertical asymptotes there.

$$\lim_{x\to 3} \frac{10x^2-90}{x^2-8x+15} = \lim_{x\to 3} \frac{10(x+3)}{x-5} = \frac{10(3+3)}{3-5} = -30$$

no vertical asymptote at $x=3$

$$\lim_{x\to 5^-} \frac{10x^2-90}{x^2-8x+15} = \lim_{x\to 5^-} \frac{10(x+3) \to 80}{x-5 \to 0^-} = -\infty$$

$f(x)$ has a vertical asymptote at $x=5$

Note: we also could have used $\lim_{x\to 5^+} f(x) = \infty$

3. (2 points each) Evaluate the following limits. For infinite limits, you must clearly show whether the limit is ∞ or $-\infty$.

(a) $\underbrace{\lim_{x\to 0+} \left(\ln(2x) - \ln\left(x^3+5x\right)\right)}$ $= \lim_{x\to 0^+} \ln\left(\frac{2x}{x^3+5x}\right)$

(indeterminate form $-\infty - (-\infty)$)

$$= \lim_{x\to 0^+} \ln\left(\frac{2x}{x(x^2+5)}\right)$$

$$= \lim_{x\to 0^+} \ln\left(\frac{2}{x^2+5}\right)$$

$$= \ln\left(\frac{2}{5}\right)$$

(b) $\lim_{x\to 0+} \frac{4e^x+5}{3-3e^x}$ $\begin{matrix}\to 9\\ \to 0^-\end{matrix}$ $= -\infty$

we used that $\lim_{x\to 0^+} (4e^x+5) = 4e^0+5 = 4+5 = 9$

and $\lim_{x\to 0^+} (3-3e^x) = 3-3e^0 = 3-3 = 0$

For $x \to 0^+$, we have $x > 0$ and $e^x > 1$

Thus $3-3e^x \to 0^-$

(c) $\lim_{x\to 10} \frac{x-10}{\sqrt{x-6}-2}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$ $= \lim_{x\to 10} \frac{x-10}{\sqrt{x-6}-2} \cdot \frac{\sqrt{x-6}+2}{\sqrt{x-6}+2}$

(indeterminate form $\frac{0}{0}$)

$$= \lim_{x\to 10} \frac{(x-10)(\sqrt{x-6}+2)}{(\sqrt{x-6}-2)(\sqrt{x-6}+2)}$$

$$= \lim_{x\to 10} \frac{(x-10)(\sqrt{x-6}+2)}{(\sqrt{x-6})^2-(2)^2}$$

$$= \lim_{x\to 10} \frac{(x-10)(\sqrt{x-6}+2)}{x-6-4}$$

$$= \lim_{x\to 10} \frac{\cancel{(x-10)}(\sqrt{x-6}+2)}{\cancel{x-10}}$$

$$= \lim_{x\to 10} (\sqrt{x-6}+2)$$

$$= \sqrt{10-6}+2$$

$$= \sqrt{4}+2$$

$$= 2+2$$

$$= 4$$