Math 220 **Quiz 2 (take-home)** **Spring 2019**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Ran Ji
- ▷ **AD2**, TR 1:00-2:50, Cassie Christenson
- ▷ **AD3**, TR 11:00-12:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Eion Blanchard
- ▷ **ADB**, TR 9:00-9:50, Eion Blanchard
- ▷ **ADC**, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- ▷ **ADD**, TR 11:00-11:50, Stathis Chrontsios
- ▷ **ADE**, TR 12:00-12:50, Kesav Krishnan
- ▷ **ADF**, TR 1:00-1:50, Souktik Roy
- ▷ **ADG**, TR 2:00-2:50, Gidon Orelowitz
- ▷ **ADH**, TR 3:00-3:50, Mina Nahvi
- ▷ **ADJ**, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- ▷ **ADK**, TR 10:00-10:50, Souktik Roy
- ▷ **ADL**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADM**, TR 12:00-12:50, Vincent Villalobos
- ▷ **ADN**, TR 1:00-1:50, Kesav Krishnan
- ▷ **ADO**, TR 2:00-2:50, Stathis Chrontsios
- ▷ **ADQ**, TR 4:00-4:50, Mina Nahvi
- ▷ **ADR**, TR 10:00-10:50, Vincent Villalobos

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write their solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your lecture period on Monday, February 4th.**
- **TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (9-9:50am, 10-10:50am).**

1. (4 points) The following two curves intersect. Find the x-value for each point of intersection.

$y = 2\ln(x+9) + x^{100}$

$y = e^{50\ln(x^2)} + \ln(3-x) + \ln(2)$

note that $e^{50\ln(x^2)} = e^{\ln((x^2)^{50})} = (x^2)^{50} = x^{100}$

$2\ln(x+9) + x^{100} = e^{50\ln(x^2)} + \ln(3-x) + \ln(2)$

$2\ln(x+9) + x^{100} = x^{100} + \ln(3-x) + \ln(2)$

$2\ln(x+9) = \ln(3-x) + \ln(2)$

$\ln((x+9)^2) = \ln((3-x)\cdot 2)$

$e^{\ln((x+9)^2)} = e^{\ln((3-x)\cdot 2)}$

$(x+9)^2 = (3-x)\cdot 2$

$x^2 + 18x + 81 = 6 - 2x$

$x^2 + 20x + 75 = 0$

$(x+5)(x+15) = 0$

$x+5 = 0 \Rightarrow x = -5$ (in domain of both functions)

$x+15 = 0 \Rightarrow x = -15$ (not in domain of first function)

The only point of intersection occurs at $x = -5$

2. (3 points) Given that the function $g(x)$ is one-to-one, determine a formula for its inverse $g^{-1}(x)$.

$$g(x) = \sqrt[3]{\frac{9-2e^{5x}}{4+e^{5x}}}$$

$$y = \sqrt[3]{\frac{9-2e^{5x}}{4+e^{5x}}}$$

$$x = \sqrt[3]{\frac{9-2e^{5y}}{4+e^{5y}}} \quad \text{(switch x \& y)}$$

$$x^3 = \frac{9-2e^{5y}}{4+e^{5y}} \quad \text{(solve for y)}$$

$$x^3(4+e^{5y}) = 9-2e^{5y}$$

$$4x^3 + x^3e^{5y} = 9-2e^{5y}$$

$$x^3e^{5y} + 2e^{5y} = 9-4x^3$$

$$(x^3+2)e^{5y} = 9-4x^3$$

$$e^{5y} = \frac{9-4x^3}{x^3+2}$$

$$\ln(e^{5y}) = \ln\left(\frac{9-4x^3}{x^3+2}\right)$$

$$5y = \ln\left(\frac{9-4x^3}{x^3+2}\right)$$

$$y = \frac{1}{5}\ln\left(\frac{9-4x^3}{x^3+2}\right)$$

$$\boxed{g^{-1}(x) = \frac{1}{5}\ln\left(\frac{9-4x^3}{x^3+2}\right)}$$

3. (3 points) Find the exponential function $f(x) = C \cdot a^x$ whose graph passes through the points $(1, 54)$ and $(4, 16)$. Be sure that both C and a are written in simplified form.

$f(1) = 54 \Rightarrow C \cdot a^1 = 54 \Rightarrow C = \frac{54}{a}$

$f(4) = 16 \Rightarrow C \cdot a^4 = 16$

Thus $\frac{54}{a} \cdot a^4 = 16$

$54a^3 = 16$

$a^3 = \frac{16}{54} = \frac{8}{27}$

$a = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

$C = \frac{54}{(2/3)} = 54 \cdot \frac{3}{2} = 81$

$f(x) = 81\left(\frac{2}{3}\right)^x$