Math 220  Quiz 12 (take-home)  Spring 2019

Name  Solutions

(circle your TA discussion section)

- AD1, TR 9:00-10:50, Ran Ji
- AD2, TR 1:00-2:50, Cassie Christenson
- AD3, TR 11:00-12:50, Dana Neidinger
- ADA, TR 8:00-8:50, Eion Blanchard
- ADB, TR 9:00-9:50, Eion Blanchard
- ADC, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- ADD, TR 11:00-11:50, Stathis Chrontsios
- ADE, TR 12:00-12:50, Kesav Krishnan
- ADF, TR 1:00-1:50, Souktik Roy
- ADG, TR 2:00-2:50, Gidon Orelowitz
- ADH, TR 3:00-3:50, Mina Nahvi
- ADJ, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- ADK, TR 10:00-10:50, Souktik Roy
- ADL, TR 11:00-11:50, Gidon Orelowitz
- ADM, TR 12:00-12:50, Vincent Villalobos
- ADN, TR 1:00-1:50, Kesav Krishnan
- ADO, TR 2:00-2:50, Stathis Chrontsios
- ADQ, TR 4:00-4:50, Mina Nahvi
- ADR, TR 10:00-10:50, Vincent Villalobos

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write their solutions separately and independently – nobody should copy someone else’s work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- The only computational technology allowed is a calculator for basic arithmetic (+, −, ×, ÷, ∨).
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available logically and legibly with sufficient work to justify each answer.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- The quiz is due at the beginning of your lecture period on Friday, April 19th.
- TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (9-9:50am, 10-10:50am).
1. (3 points) Suppose that the polynomial \( g(x) \) satisfies the following conditions.

- \( g(x) \) is an odd function
- \( g(5) = 9 \)
- \( g'(5) = 3 \)
- \( g''(5) = 2 \)
- \( g'''(5) = 1 \)

Use the techniques of linear approximation found in section 3.10 to estimate the value of \( g(-5.02) \). Simplify and write your answer in decimal form.

\[
\text{point: } (5, g(5)) = (5, 9) \\
\text{slope: } g'(5) = 3 \\
\text{tangent line at } x=5: y-9 = 3(x-5) \\
y = 9 + 3(x-5) \\
g(x) \approx 9 + 3(x-5) \text{ for } x \text{ near } 5 \\
g(5.02) \approx 9 + 3(5.02-5) \\
g(5.02) \approx 9.06 \\
g(-5.02) \approx -g(5.02) \text{ since } g(x) \text{ is odd} \\
g(-5.02) \approx -9.06
\]
2. (3 points) Express \(2 \ln (4e^2) - 2 \ln (2) - \ln (5e^4)\) as a single logarithm. Now use the techniques of linear approximation found in section 3.10 to estimate its value. Simplify and write your answer in decimal form.

\[
2 \ln (4e^2) - 2 \ln (2) - \ln (5e^4) = \\
\ln (4e^2) - \ln (2) - \ln (5e^4) = \\
\ln (16e^4) - \ln (4) - \ln (5e^4) = \\
\ln \left( \frac{16e^4}{4} \right) - \ln (5e^4) = \\
\ln (4e^4) - \ln (5e^4) = \\
\ln \left( \frac{4e^4}{5e^4} \right) = \ln \left( \frac{4}{5} \right)
\]

Let \(f(x) = \ln (x)\) and find its tangent line at \(x = 1\).

Point: \((1, f(1)) = (1, \ln (1)) = (1, 0)\)

Slope: \(f'(1) = \frac{1}{1} = 1\) (since \(f'(x) = \frac{1}{x}\))

Tangent line: \(y = 0 = 1 \cdot (x - 1) \Rightarrow y = x - 1\)

\[\ln (x) \approx x - 1 \text{ for } x \text{ near } 1\]

\(\ln (4/5) \approx -1/5\)

\(\ln (4/5) \approx -0.2\)
3. (4 points) There is one value of $x$ for which the line tangent to the graph of $f(x) = x^4 + x^2$ is perpendicular to the line $y = \frac{1}{20}x - 10$. Determine this $x$-value using Newton’s Method with an initial estimate of $x_1 = -1$. You should use this method 3 times in order to obtain estimates $x_2$, $x_3$ and $x_4$. You are only allowed to use technology for basic arithmetic. Your final answer should include 5 places after the decimal point.

The line $y = \frac{1}{20}x - 10$ has slope $\frac{1}{20}$

Each line perpendicular to this line has slope $\frac{-1}{\frac{1}{20}} = -20$

Each line tangent to $f(x) = x^4 + x^2$ at $x$ has slope $f'(x) = 4x^3 + 2x$

Thus $4x^3 + 2x = -20$

or $4x^3 + 2x + 20 = 0$

Let $g(x) = 4x^3 + 2x + 20$ and apply Newton’s method to estimate a root of $g(x)$

$g'(x) = 12x^2 + 2$

$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$

$x_1 = -1$

$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = -1 - \frac{g(-1)}{g'(-1)} = -1 - \frac{14}{14} = -2$

$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = -2 - \frac{g(-2)}{g'(-2)} = -2 - \frac{16}{50} = -\frac{42}{25} = -1.68$

$x_4 = x_3 - \frac{g(x_3)}{g'(x_3)} = -1.68 - \frac{g(-1.68)}{g'(-1.68)}$

$\approx -1.615137836$

$\approx -1.61514$