Math 220 | Quiz 12 (take-home) | Spring 2019

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Ran Ji
- ▷ **AD2**, TR 1:00-2:50, Cassie Christenson
- ▷ **AD3**, TR 11:00-12:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Eion Blanchard
- ▷ **ADB**, TR 9:00-9:50, Eion Blanchard
- ▷ **ADC**, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- ▷ **ADD**, TR 11:00-11:50, Stathis Chrontsios
- ▷ **ADE**, TR 12:00-12:50, Kesav Krishnan
- ▷ **ADF**, TR 1:00-1:50, Souktik Roy
- ▷ **ADG**, TR 2:00-2:50, Gidon Orelowitz
- ▷ **ADH**, TR 3:00-3:50, Mina Nahvi
- ▷ **ADJ**, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- ▷ **ADK**, TR 10:00-10:50, Souktik Roy
- ▷ **ADL**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADM**, TR 12:00-12:50, Vincent Villalobos
- ▷ **ADN**, TR 1:00-1:50, Kesav Krishnan
- ▷ **ADO**, TR 2:00-2:50, Stathis Chrontsios
- ▷ **ADQ**, TR 4:00-4:50, Mina Nahvi
- ▷ **ADR**, TR 10:00-10:50, Vincent Villalobos

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write their solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- The only computational technology allowed is a calculator for basic arithmetic ($+$, $-$, $\times$, $\div$, $\wedge$).
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your lecture period on Friday, April 19th.**
- **TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (9-9:50am, 10-10:50am).**

1. (3 points) Suppose that the polynomial $g(x)$ satisfies the following conditions.

- $g(x)$ is an odd function
- $g(5) = 9$
- $g'(5) = 3$
- $g''(5) = 2$
- $g'''(5) = 1$

Use the techniques of linear approximation found in section 3.10 to estimate the value of $g(-5.02)$. Simplify and write your answer in decimal form.

point: $(5, g(5)) = (5, 9)$

slope: $g'(5) = 3$

tangent line at $x=5$: $y - 9 = 3(x-5) \Rightarrow$

$y = 9 + 3(x-5)$

$g(x) \approx 9 + 3(x-5)$ for x near 5

$g(5.02) \approx 9 + 3(5.02 - 5)$

$g(5.02) \approx 9.06$

$g(-5.02) = -g(5.02)$ since $g(x)$ is odd

$g(-5.02) \approx -9.06$

2. (3 points) Express $2\ln\left(4e^2\right) - 2\ln\left(2\right) - \ln\left(5e^4\right)$ as a single logarithm. Now use the techniques of linear approximation found in section 3.10 to estimate its value. Simplify and write your answer in decimal form.

$$2\ln(4e^2) - 2\ln(2) - \ln(5e^4) =$$

$$\ln((4e^2)^2) - \ln(2^2) - \ln(5e^4) =$$

$$\ln(16e^4) - \ln(4) - \ln(5e^4) =$$

$$\ln\left(\frac{16e^4}{4}\right) - \ln(5e^4) =$$

$$\ln(4e^4) - \ln(5e^4) =$$

$$\ln\left(\frac{4e^4}{5e^4}\right) = \ln\left(\frac{4}{5}\right)$$

Let $f(x) = \ln(x)$ and find its tangent line at $x = 1$

Point: $(1, f(1)) = (1, \ln(1)) = (1, 0)$

slope: $f'(1) = \frac{1}{1}$ (since $f'(x) = \frac{1}{x}$)

tangent line: $y - 0 = 1 \cdot (x - 1) \Rightarrow y = x - 1$

$\ln(x) \approx x - 1$ for x near 1

$$\ln\left(\tfrac{4}{5}\right) \approx \tfrac{4}{5} - 1$$

$$\ln\left(\tfrac{4}{5}\right) \approx -\tfrac{1}{5}$$

$$\ln\left(\tfrac{4}{5}\right) \approx -0.2$$

3. (4 points) There is one value of x for which the line tangent to the graph of $f(x) = x^4 + x^2$ is perpendicular to the line $y = \frac{1}{20}x - 10$. Determine this x-value using Newton's Method with an initial estimate of $x_1 = -1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Your final answer should include 5 places after the decimal point.

- The line $y = \frac{1}{20}x - 10$ has slope $\frac{1}{20}$
- Each line perpendicular to this line has slope $\frac{-1}{1/20} = -20$
- Each line tangent to $f(x) = x^4 + x^2$ at x has slope $f'(x) = 4x^3 + 2x$

Thus $4x^3 + 2x = -20$

or $4x^3 + 2x + 20 = 0$

Let $g(x) = 4x^3 + 2x + 20$ and apply Newton's method to estimate a root of $g(x)$

$g'(x) = 12x^2 + 2$

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$$

$x_1 = -1$

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = -1 - \frac{g(-1)}{g'(-1)} = -1 - \frac{14}{14} = -2$$

$$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = -2 - \frac{g(-2)}{g'(-2)} = -2 - \frac{-16}{50} = \frac{-42}{25} = -1.68$$

$$x_4 = x_3 - \frac{g(x_3)}{g'(x_3)} = -1.68 - \frac{g(-1.68)}{g'(-1.68)}$$

$$= -1.68 - \frac{-2.326528}{35.8688}$$

$$\approx -1.615137836$$

$$\approx -1.61514$$