Math 220 (section AD?) **Quiz 11** **Spring 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = \dfrac{240}{\sqrt{8x+9}}$ on the interval $[-1, 2]$. Simplify your answer.

$$f_{ave} = \frac{1}{2-(-1)} \int_{-1}^{2} \frac{240}{\sqrt{8x+9}}\,dx$$

$$= \frac{1}{3} \int_{-1}^{2} \frac{30}{\sqrt{8x+9}} \cdot 8dx$$

$$= \frac{30}{3} \int_{1}^{25} \frac{1}{\sqrt{u}}\,du$$

$$= 10 \int_{1}^{25} u^{-1/2}\,du$$

$$= 10 \cdot \frac{u^{1/2}}{1/2} \Big|_{1}^{25}$$

$$= 20\sqrt{u} \Big|_{1}^{25}$$

$$= 20\sqrt{25} - 20\sqrt{1}$$

$$= 20 \cdot 5 - 20 \cdot 1$$

$$= 80$$

$$\boxed{\begin{array}{l} \underline{u\text{-substitution}} \\ u = 8x+9 \\ du = 8dx \\ \hline x=-1 \Rightarrow u = 8(-1)+9 \\ \qquad = 1 \\ \hline x = 2 \Rightarrow u = 8(2)+9 \\ \qquad = 25 \end{array}}$$

2. Let **R** be the finite region bounded by the graphs of the following equations.

$$y = \frac{100}{x^2}$$

$$y = 25$$

$$x = 5$$

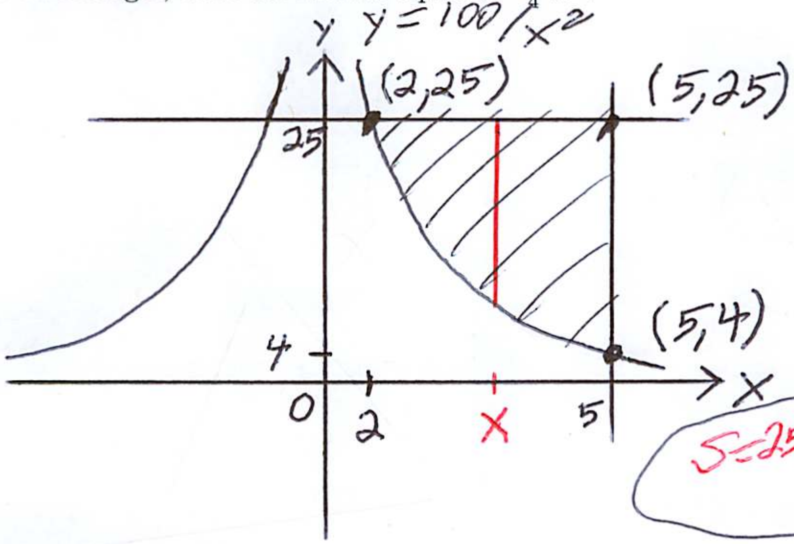
intersections

$25 = \frac{100}{x^2} \Rightarrow 25x^2 = 100 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$x = 5 \Rightarrow y = \frac{100}{5^2} = \frac{100}{25} = 4$

Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

(a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are equilateral triangles. Hint: If S represents the length of each side in an equilateral triangle, then its area is equal to $\frac{\sqrt{3}}{4}S^2$.



cross-section at x is an equilateral triangle

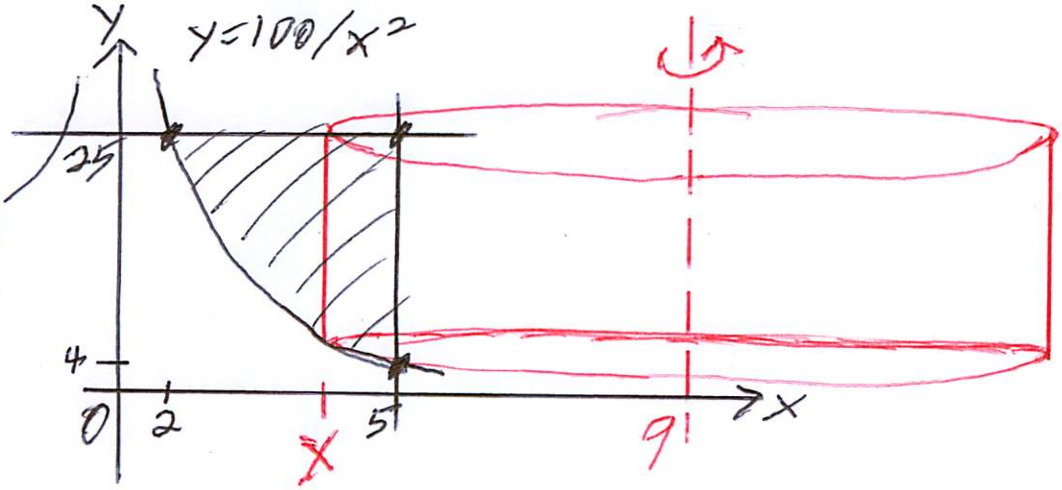
$$V = \int_{x_{min}}^{x_{max}} A(x)\,dx$$ where $A(x)$ is the cross-sectional area of a slice at x

$$V = \int_2^5 \frac{\sqrt{3}}{4} S^2\,dx$$

$$V = \int_2^5 \frac{\sqrt{3}}{4}\left(25 - \frac{100}{x^2}\right)^2 dx$$

(b) The volume of the solid formed when **R** is revolved around the line $x = 9$. Set up the integrals for this volume in the following two ways.

i. (2 points) Integrate with respect to x.



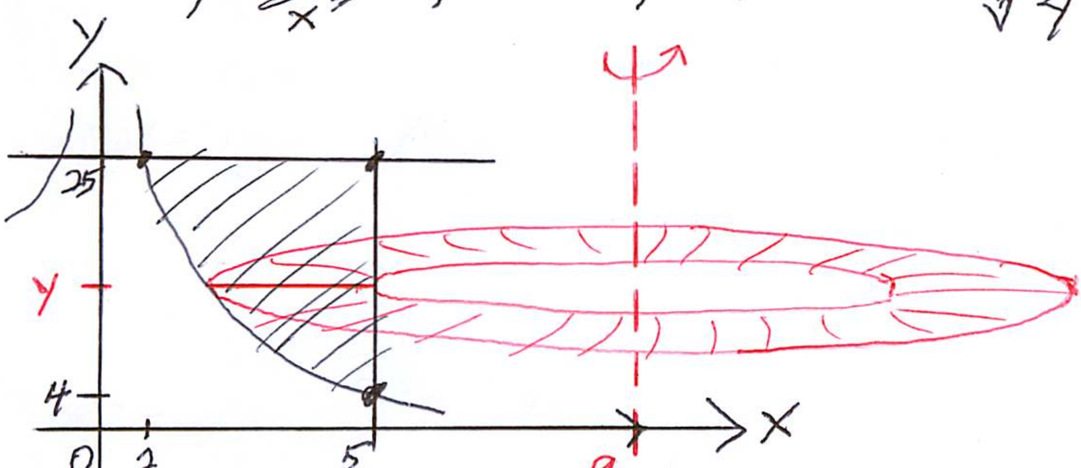
$$V = \int_{x_{min}}^{x_{max}} (\text{surface area})\, dx$$

$$V = \int_2^5 2\pi r h \, dx$$

$$V = \int_2^5 2\pi (9-x)\left(25 - \frac{100}{x^2}\right) dx$$

ii. (2 points) Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)

$$y = \frac{100}{x^2} \Rightarrow x^2 = \frac{100}{y} \Rightarrow x = \pm\sqrt{\frac{100}{y}} \quad \left(\text{use } x = \sqrt{\frac{100}{y}} \text{ in first quadrant}\right)$$



$$V = \int_{y_{min}}^{y_{max}} (\text{cross-sectional area})\, dy$$

$$V = \int_4^{25} \left(\pi r_{out}^2 - \pi r_{in}^2\right) dy$$

$$V = \int_4^{25} \left(\pi\left(9 - \sqrt{\frac{100}{y}}\right)^2 - \pi(9-5)^2\right) dy$$