Math 220 (section AD?) **Quiz 10** **Spring 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Fill in the missing information for the following two theorems.

$\boxed{\textbf{Rolle's Theorem}}$ Let f be a function that satisfies the following three hypotheses.

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

(3) $f(a) = f(b)$.

Then there is a number c in (a, b) such that $f'(c) = 0$.

$\boxed{\textbf{Mean Value Theorem}}$ Let f be a function that satisfies the following two hypotheses.

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_1^2 \frac{-180x^2}{(x^3+2)^2}\,dx = \int_1^2 \frac{-180}{(x^3+2)^2}\cdot x^2\,dx$$

$u = x^3+2$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$x=1 \Rightarrow u = 1^3+2=3$
$x=2 \Rightarrow u = 2^3+2=10$

$$= \int_3^{10} \frac{-180}{u^2}\cdot\frac{1}{3}\,du$$
$$= -60\int_3^{10} u^{-2}\,du$$
$$= -60\left[\frac{u^{-1}}{-1}\right]_3^{10}$$
$$= -60\left[\frac{-1}{u}\right]_3^{10}$$
$$= -60\left[\frac{-1}{10} - \frac{-1}{3}\right]$$
$$= 6 - 20 = -14$$

3. (2 points) Evaluate the indefinite integral.

$$\int \frac{15x^9}{\sqrt{x^5+2}}\,dx = \int \frac{3x^5}{\sqrt{x^5+2}}\cdot 5x^4\,dx$$

$u = x^5+2$
$du = 5x^4\,dx$
$u-2 = x^5$

$$= \int \frac{3(u-2)}{\sqrt{u}}\,du$$
$$= \int\left(\frac{3u}{\sqrt{u}} - \frac{6}{\sqrt{u}}\right)du$$
$$= \int (3u^{1/2} - 6u^{-1/2})\,du$$
$$= 3\frac{u^{3/2}}{3/2} - 6\frac{u^{1/2}}{1/2} + C$$
$$= 2u^{3/2} - 12u^{1/2} + C$$
$$= 2(x^5+2)^{3/2} - 12(x^5+2)^{1/2} + C$$

4. (2 points) Evaluate the indefinite integral.

$$\int 100e^{5x}\sin\left(e^{5x}\right)\cos^9\left(e^{5x}\right)dx = \int 20\sin(e^{5x})\cos^9(e^{5x})\cdot 5e^{5x}dx$$

$$\boxed{\begin{aligned} u &= e^{5x} \\ du &= 5e^{5x}dx \end{aligned}}$$

$$= \int 20\sin(u)\cos^9(u)\,du$$

$$= \int \cos^9(u)\cdot 20\sin(u)\,du$$

$$\boxed{\begin{aligned} w &= \cos(u) \\ dw &= -\sin(u)du \\ -20dw &= 20\sin(u)du \end{aligned}}$$

$$= \int w^9\cdot(-20dw)$$

$$= -20\int w^9 dw$$

$$= -20\cdot\frac{w^{10}}{10} + C$$

$$= -2w^{10} + C$$

$$= -2\cos^{10}(u) + C$$

$$= -2\cos^{10}(e^{5x}) + C$$

Alternate approach

$$\boxed{\begin{aligned} u &= \cos(e^{5x}) \\ du &= -\sin(e^{5x})\cdot 5e^{5x}dx \\ -\tfrac{1}{5}du &= \sin(e^{5x})e^{5x}dx \end{aligned}}$$

$$\int 100e^{5x}\sin(e^{5x})\cos^9(e^{5x})dx =$$

$$100\int \cos^9(e^{5x})\sin(e^{5x})e^{5x}dx =$$

$$100\int u^9\cdot -\tfrac{1}{5}du =$$

$$-20\int u^9 du =$$

$$-20\cdot\frac{u^{10}}{10} + C =$$

$$-2u^{10} + C =$$

$$-2\cos^{10}(e^{5x}) + C$$

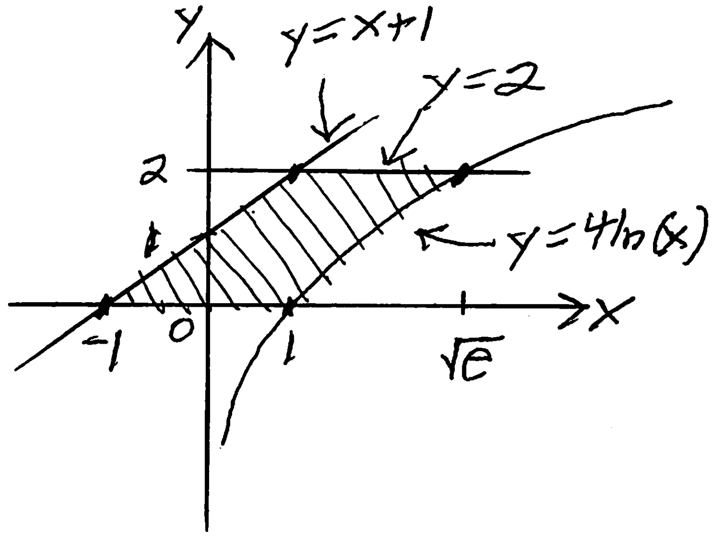
5. (2 points) Let **R** be the finite region bounded by the given functions. In the following way, set up <u>but do not evaluate</u> definite integrals which represent the area of the region **R**.

$y = 4\ln(x)$

$y = 0$

$y = 2$

$y = x + 1$



intersection of $y=4\ln(x)$ and $y=2$	intersection of $y=x+1$ and $y=2$
$4\ln(x)=2$	$x+1=2$
$\ln(x)=\frac{2}{4}=\frac{1}{2}$	$x=1$
$x=e^{1/2}=\sqrt{e}$	

(a) Integrate with respect to x.

$$\text{area} = \int_{x_{min}}^{x_{max}} (y_{top} - y_{bottom})\,dx$$

$$\text{area} = \int_{-1}^{1} (x+1-0)\,dx + \int_{1}^{\sqrt{e}} (2 - 4\ln(x))\,dx$$

note) top boundary changes at $x=1$
and bottom boundary also changes at $x=1$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$\text{area} = \int_{y_{min}}^{y_{max}} (x_{right} - x_{left})\,dy$$

$$\text{area} = \int_{0}^{2} \left(e^{y/4} - (y-1)\right) dy$$

note) $y=4\ln(x) \Rightarrow x = e^{y/4}$ for right boundary
$y=x+1 \Rightarrow x=y-1$ for left boundary