AL1 (MWF 11:00-11:50) AL2 (MWF 1:00-1:50)

MATH 220 **Test 2** **Spring 2017**

Name Solutions

NetID ____________________

UIN ____________________

Circle your TA discussion section.

- ▷ **AD1**, TR 9:00-10:50, Andrew McConvey
- ▷ **AD2**, TR 1:00-2:50, Sarah Loeb
- ▷ **ADA**, TR 8:00-8:50, Christopher Linden
- ▷ **ADB**, TR 9:00-9:50, Dakota Ihli
- ▷ **ADC**, TR 10:00-10:50, Cassie Christenson
- ▷ **ADD**, TR 11:00-11:50, Daulet Dyussekenov
- ▷ **ADE**, TR 12:00-12:50, Daulet Dyussekenov
- ▷ **ADF**, TR 1:00-1:50, Cassie Christenson
- ▷ **ADG**, TR 2:00-2:50, Xinghua Gao
- ▷ **ADH**, TR 3:00-3:50, Xinghua Gao
- ▷ **ADJ**, TR 9:00-9:50, Lan Wang
- ▷ **ADK**, TR 10:00-10:50, Lan Wang
- ▷ **ADO**, TR 2:00-2:50, Christopher Linden
- ▷ **ADQ**, TR 4:00-4:50, Dakota Ihli

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

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FRONT OF ROOM – 100 Materials Science and Engineering Building

1. (10 points) Find $g'(t)$ given that $g(t) = \frac{\arctan(t^9)}{t^{21}}$

$$g'(t) = \frac{\frac{d}{dt}(\arctan(t^9))\cdot t^{21} - \arctan(t^9)\cdot\frac{d}{dt}(t^{21})}{(t^{21})^2}$$

$$= \boxed{\frac{\frac{1}{1+(t^9)^2}\cdot 9t^8\cdot t^{21} - \arctan(t^9)\cdot 21t^{20}}{(t^{21})^2}}$$

or use product rule

on $g(t) = \arctan(t^9)\cdot t^{-21}$

2. (10 points) Find $w'(x)$ given that $w(x) = e^{7\sin^6(5x)}$

$$w'(x) = e^{7\sin^6(5x)}\cdot\frac{d}{dx}(7\sin^6(5x))$$

$$= e^{7\sin^6(5x)}\cdot 7\cdot 6\sin^5(5x)\cdot\frac{d}{dx}(\sin(5x))$$

$$\boxed{= e^{7\sin^6(5x)}\cdot 7\cdot 6\sin^5(5x)\cdot\cos(5x)\cdot 5}$$

3. (10 points) Find $\frac{dy}{dx}$ given that $x^5y^8 = 14x^3 + 5y$

$$\frac{d}{dx}(x^5y^8) = \frac{d}{dx}(14x^3+5y)$$

$$\frac{d}{dx}(x^5)\cdot y^8 + x^5\cdot\frac{d}{dx}(y^8) = 42x^2 + 5\frac{dy}{dx}$$

$$5x^4\cdot y^8 + x^5\cdot 8y^7\frac{dy}{dx} = 42x^2 + 5\frac{dy}{dx}$$

$$8x^5y^7\frac{dy}{dx} - 5\frac{dy}{dx} = 42x^2 - 5x^4y^8$$

$$\frac{dy}{dx}(8x^5y^7 - 5) = 42x^2 - 5x^4y^8$$

$$\boxed{\frac{dy}{dx} = \frac{42x^2 - 5x^4y^8}{8x^5y^7 - 5}}$$

4. (10 points) Find the equation of the line tangent to the following curve at its y-intercept.

$y = 5\sin(x) + 25e^x + 12x + 10$

For y-int, set $x=0$.

$$y = 5\sin(0) + 25e^0 + 12(0) + 10$$
$$= 35$$

$\boxed{\text{Point: } (0,35)}$

$$y' = 5\cos(x) + 25e^x + 12$$

$$y'(0) = 5\cos(0) + 25e^0 + 12$$
$$= 42$$

$\boxed{\text{Slope: } 42}$

$\boxed{\text{tangent line: } y = 42x + 35}$

5. (10 points) A bullet is shot upward from the surface of a planet so that its height in meters until coming to rest is given by the equation $s(t) = 195t - 6.5t^2$ where t is measured in seconds. Answer the following questions and be sure to use proper units in each answer.

(a) What is the acceleration due to gravity on this planet?

(vel) $s'(t) = 195 - 13t$

$\boxed{\text{(acc) } s''(t) = -13 \text{ m/s}^2}$

(b) What is the bullet's initial velocity?

$\boxed{s'(0) = 195 \text{ m/s}}$

(c) At what time does the bullet reach its maximum height?

crit. number) $s'(t) = 0 \Rightarrow 195 - 13t = 0 \Rightarrow t = \frac{195}{13} = 15$

values of $s'(t)$ $\quad \underset{}{+++} \overset{0}{\underset{15}{|}} --- \to t$

$\boxed{\text{max. height at } t = 15 \text{ s}}$

6. (10 points) Evaluate the limit. You must fully justify your answer.

$$\lim_{x\to 0} \frac{e^{13x} - 13x - 1 \ \nearrow 0}{e^{7x} - 7x - 1 \ \searrow 0} \overset{H}{=} \lim_{x\to 0} \frac{13e^{13x} - 13 \ \nearrow 0}{7e^{7x} - 7 \ \to 0}$$

(by l'Hospital's Rule)

$$\overset{H}{=} \lim_{x\to 0} \frac{169e^{13x}}{49e^{7x}}$$

(by l'Hospital's Rule)

$$= \boxed{\frac{169}{49}}$$

7. (10 points) Find the absolute minimum y-value of the given function.

$$y = \frac{2x}{\sqrt{x-81}}$$ domain: $x-81>0 \Rightarrow x>81$

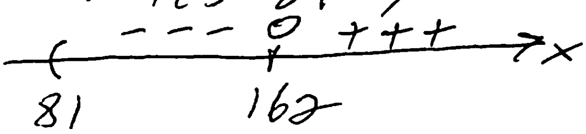
$$y' = \frac{2\cdot\sqrt{x-81} - 2x\cdot\frac{1}{2}(x-81)^{-1/2}}{(\sqrt{x-81})^2}$$

$$= \frac{2\sqrt{x-81} - \frac{x}{\sqrt{x-81}}}{x-81} \quad \left(\text{multiply by } \frac{\sqrt{x-81}}{\sqrt{x-81}}\right)$$

$$= \frac{2(x-81)-x}{(x-81)\sqrt{x-81}}$$

$$= \frac{x-162}{(x-81)\sqrt{x-81}}$$

values of y'



abs. min at $x=162$ is

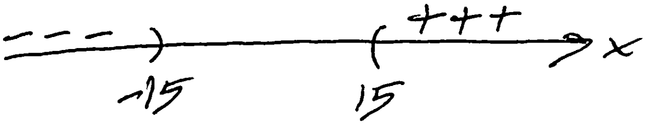
$$y = \frac{2\cdot 162}{\sqrt{162-81}} = \frac{324}{\sqrt{81}} = \frac{324}{9} = 36$$

8. (10 points) Let $f(x) = \ln(x^2-225)$. Determine each interval where f is increasing and each interval where f is decreasing.

domain; $x^2-225>0 \Rightarrow x^2>225 \Rightarrow |x|>15 \Rightarrow x<-15$ or $x>15$

$$f'(x) = \frac{1}{x^2-225}\cdot 2x = \frac{2x}{(x+15)(x-15)}$$

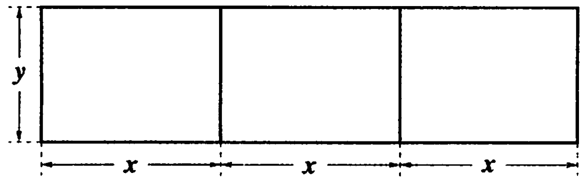
values of $f'(x)$



$f'<0 \Rightarrow f$ decreasing on $(-\infty,-15)$

$f'>0 \Rightarrow f$ increasing on $(15,\infty)$

9. (10 points) A farmer wishes to fence off three identical adjoining rectangular pens as in the diagram shown, but she only has 600 feet of fencing available. Determine the values for x and y which will maximize the total area enclosed by these three pens.



$$\left(\begin{aligned} 6x+4y&=600\\ 4y&=600-6x\\ y&=\frac{600-6x}{4}\\ y&=150-\tfrac{3}{2}x\end{aligned}\right)$$

$$\begin{aligned} A&=3xy\\ &=3x(150-\tfrac{3}{2}x)\\ &=450x-\tfrac{9}{2}x^2\end{aligned}$$

maximize A for x in $(0,100)$ (do you see why $x<100$?)

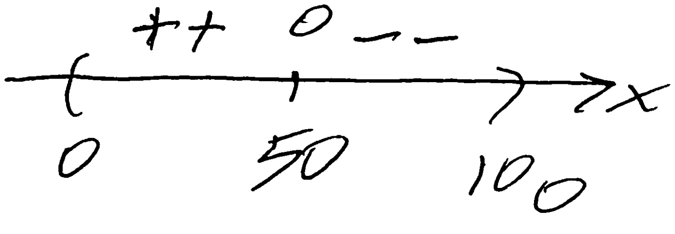
$$A'=450-9x$$

$$\begin{aligned} 0&=450-9x\\ 9x&=450\\ x&=50\end{aligned}$$

abs max. at $x=50$

$$\begin{aligned} y&=150-\tfrac{3}{2}(50)\\ &=75\end{aligned}$$

values of A'



$x=50$ ft

$y=75$ ft

$A=11250$ ft^2

10. (10 points) A spherical balloon is inflated at a constant rate of 120π cubic feet per minute. At one particular time, the balloon's radius is increasing at 5 feet per minute. What is the balloon's radius at that particular time?

Given: $\frac{dV}{dt} = 120\pi \text{ ft}^3/\text{min}$

want: $r\Big|_{\frac{dr}{dt} = 5 \text{ ft/min}}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$120\pi = 4\pi r^2(5)$$

$$\frac{120\pi}{20\pi} = r^2$$

$$6 = r^2$$

$$\boxed{r = \sqrt{6} \text{ ft}}$$

Students – do not write on this page!

1. (10 points) ____________________

2. (10 points) ____________________

3. (10 points) ____________________

4. (10 points) ____________________

5. (10 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8. (10 points) ____________________

9. (10 points) ____________________

10. (10 points) ____________________

TOTAL (100 points) ______________