Math 220 (section AD?) Quiz 9 Spring 2017

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{3x^2}{x^2+1}\,dx = \int \frac{3x^2+3-3}{x^2+1}\,dx$

$= \int \frac{3(x^2+1)-3}{x^2+1}\,dx$

$= \int \left(\frac{3(x^2+1)}{x^2+1} - \frac{3}{x^2+1}\right)dx$

$= \int \left(3 - 3\cdot\frac{1}{x^2+1}\right)dx$

$= 3x - 3\arctan(x) + C$

or use polynomial long division to get $\frac{3x^2}{x^2+1} = 3 + \frac{-3}{x^2+1}$

(b) $\int \left(\cos(2x) + 2\sin^2(x) + \tan^2(x)\right)\,dx$

$= \int \left(\cos^2(x) - \sin^2(x) + 2\sin^2(x) + \tan^2(x)\right)dx$

$= \int \left(\cos^2(x) + \sin^2(x) + \tan^2(x)\right)dx$

$= \int \left(1 + \tan^2(x)\right)dx$

$= \int \sec^2(x)\,dx$

$= \tan(x) + C$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_{-5}^{-1} \frac{3x - 2x^2}{x^2}\, dx = \int_{-5}^{-1} \left(\frac{3x}{x^2} - \frac{2x^2}{x^2} \right) dx$$

$$= \int_{-5}^{-1} \left(\frac{3}{x} - 2 \right) dx$$

$$= \Big[3\ln|x| - 2x \Big]_{-5}^{-1}$$

$$= [3\ln|-1| - 2(-1)] - [3\ln|-5| - 2(-5)]$$

$$= [3 \cdot 0 + 2] - [3\ln(5) + 10]$$

$$= -3\ln(5) - 8$$

3. (2 points) Suppose $g(x) = \int_{x^2}^{3} \frac{1}{t^4+1}\, dt$. Find its second derivative $g''(x)$.

$$g(x) = -\int_{3}^{x^2} \frac{1}{t^4+1}\, dt$$

From part 1 of the Fundamental Theorem of Calculus along with the Chain Rule,

$$g'(x) = -\frac{1}{(x^2)^4+1} \cdot \frac{d}{dx}(x^2)$$

$$g'(x) = \frac{-2x}{x^8+1}$$

$$g''(x) = \frac{\frac{d}{dx}(-2x)\cdot(x^8+1) - (-2x)\cdot\frac{d}{dx}(x^8+1)}{(x^8+1)^2}$$

$$g''(x) = \frac{-2(x^8+1) + 2x \cdot 8x^7}{(x^8+1)^2} = \frac{14x^8 - 2}{(x^8+1)^2}$$

4. (2 points) The height of a tree is currently 12 inches. If the tree's height increases by $4\sqrt[3]{t}$ inches per year where t is measured in years from now, then what will the tree's height be in 8 years?

$$\begin{pmatrix}\text{height} \\ \text{at } t=8\end{pmatrix} = \begin{pmatrix}\text{height} \\ \text{at } t=0\end{pmatrix} + \begin{pmatrix}\text{net} \\ \text{change in height} \\ \text{between } t=0 \text{ and } t=8\end{pmatrix}$$

$$= 12 + \int_0^8 \begin{pmatrix}\text{rate of change} \\ \text{in height}\end{pmatrix} dt$$

$$= 12 + \int_0^8 4\sqrt[3]{t}\, dt$$

$$= 12 + \int_0^8 4t^{1/3}\, dt$$

$$= 12 + \left[4 \cdot \frac{1}{4/3} t^{4/3}\right]_0^8$$

$$= 12 + \left[3t^{4/3}\right]_0^8$$

$$= 12 + \left[3(8)^{4/3} - 3(0)^{4/3}\right]$$

$$= 12 + \left[3 \cdot 16 - 3 \cdot 0\right]$$

$$= 12 + 48$$

$$= 60 \text{ inches (or 5 ft)}$$