Math 220 (section AD?) Quiz 6 Spring 2017

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) For $t \geq 0$, the position in meters of a particle is given by

$$s(t) = \frac{t^3}{3} - t^2 + 2t + 9$$

where t is measured in seconds.

What is the particle's acceleration at the moment when the particle's velocity is 10 m/s ? Use correct units in your final answer.

position: $s(t) = \frac{1}{3}t^3 - t^2 + 2t + 9$

velocity: $s'(t) = t^2 - 2t + 2$

acceleration: $s''(t) = 2t - 2$

$s'(t) = 10$

$t^2 - 2t + 2 = 10$

$t^2 - 2t - 8 = 0$

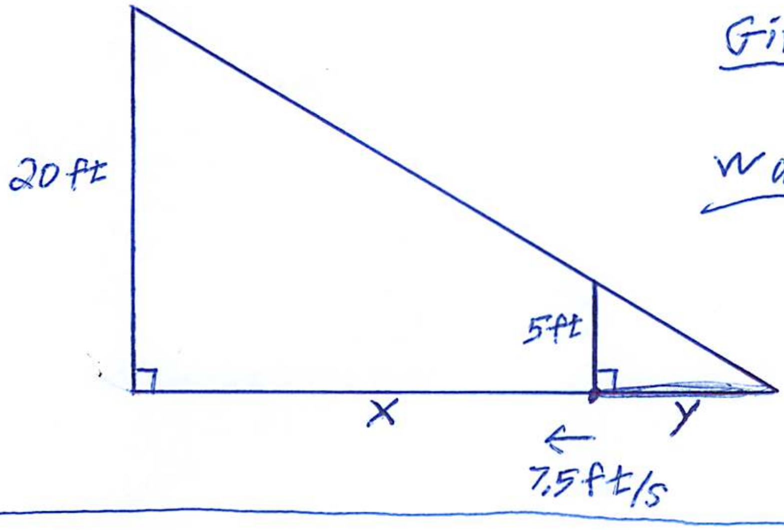
$(t-4)(t+2) = 0$

$t \geq 0$ so $t = 4$

$s''(4) = 2 \cdot 4 - 2$

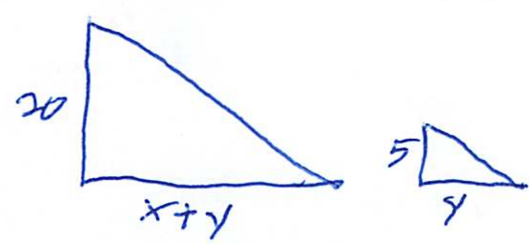
$= 6 \ m/s^2$

2. (3 points) A street light is mounted on top of a 20 ft tall pole. A woman 5 ft tall walks directly toward the pole with a speed of 7.5 ft/s. How quickly is the length of her shadow decreasing when she is 13 ft from the pole?



Given) $\frac{dx}{dt} = -7.5\ ft/s$

want) $\left.\frac{dy}{dt}\right|_{x=13\,ft}$



From similar triangles,

$$\frac{x+y}{20} = \frac{y}{5} \Rightarrow 20y = 5(x+y)$$
$$4y = x+y$$
$$3y = x$$
$$y = \frac{1}{3}x$$

$$\frac{d}{dt}(y) = \frac{d}{dt}\left(\frac{1}{3}x\right)$$

$$\frac{dy}{dt} = \frac{1}{3}\frac{dx}{dt}$$

$$\left.\frac{dy}{dt}\right|_{x=13\,ft} = \frac{1}{3}(-7.5) = -2.5\ ft/s$$

The length of her shadow is decreasing at 2.5 ft/s

Note that her shadow's length decreases at a constant rate, Her distance from the pole is not needed

3. (4 points) A bacteria culture grows with a constant relative growth rate. The initial bacteria count of 100 increases to 150 after 2 hours.

(a) Find an expression for the number of bacteria after t hours.

rel. growth rate $\frac{dP/dt}{P} = k$, a constant

$\frac{dP}{dt} = kP \Rightarrow P = Ce^{kt}$

$P(0) = 100 \Rightarrow 100 = Ce^{k \cdot 0} \Rightarrow C = 100$

$P = 100e^{kt}$

$P(2) = 150 \Rightarrow 150 = 100e^{k \cdot 2}$

$1.5 = e^{2k}$

$\ln(1.5) = 2k$

$k = \frac{1}{2}\ln(1.5)$

$P = 100e^{\frac{1}{2}\ln(1.5)t}$

(b) At what time will the bacteria count reach 800 ?

$800 = 100e^{\frac{1}{2}\ln(1.5)t}$

$8 = e^{\frac{1}{2}\ln(1.5)t}$

$\ln(8) = \frac{1}{2}\ln(1.5)t$

$t = \frac{2\ln(8)}{\ln(1.5)}$ hours