Math 220 (section AD?) Quiz 5 Spring 2017

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Compute the first derivative $w'(x)$ for the given function.

$$w(x) = \cot^3\left(e^{5x}\right)$$

$w(x) = \left(\cot(e^{5x})\right)^3$

$w'(x) = 3\left(\cot(e^{5x})\right)^2 \cdot \frac{d}{dx}\left(\cot(e^{5x})\right)$

$w'(x) = 3\cot^2(e^{5x}) \cdot -\csc^2(e^{5x}) \cdot \frac{d}{dx}(e^{5x})$

$w'(x) = 3\cot^2(e^{5x}) \cdot -\csc^2(e^{5x}) \cdot e^{5x} \cdot \frac{d}{dx}(5x)$

$w'(x) = 3\cot^2(e^{5x}) \cdot -\csc^2(e^{5x}) \cdot e^{5x} \cdot 5$

$w'(x) = -15\cot^2(e^{5x})\csc^2(e^{5x})e^{5x}$

2. (2 points) Compute the second derivative $P''(t)$ for the given function.

$$P(t) = \arctan\left(5t^2\right)$$

$$P'(t) = \frac{1}{1+(5t^2)^2} \cdot \frac{d}{dt}(5t^2)$$

$$P'(t) = \frac{1}{1+25t^4} \cdot 10t$$

$$P'(t) = \frac{10t}{1+25t^4}$$

$$P''(t) = \frac{\frac{d}{dt}(10t)\cdot(1+25t^4) - 10t\cdot\frac{d}{dt}(1+25t^4)}{(1+25t^4)^2}$$

$$P''(t) = \frac{10(1+25t^4) - 10t(100t^3)}{(1+25t^4)^2}$$

$$P''(t) = \frac{10 - 750t^4}{(1+25t^4)^2}$$

3. (3 points) Find the equation of the line tangent to the given curve at the point $(2, -1)$.

$$y^3 + x^2 = x^3y^2 - 5$$

$$\frac{d}{dx}(y^3+x^2) = \frac{d}{dx}(x^3y^2-5)$$

$$3y^2 \cdot \frac{dy}{dx} + 2x = \frac{d}{dx}(x^3) \cdot y^2 + x^3 \cdot \frac{d}{dx}(y^2) - 0$$

$$3y^2 \cdot \frac{dy}{dx} + 2x = 3x^2 \cdot y^2 + x^3 \cdot 2y\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 2x^3y\frac{dy}{dx} = 3x^2y^2 - 2x$$

$$\frac{dy}{dx}(3y^2 - 2x^3y) = 3x^2y^2 - 2x$$

$$\frac{dy}{dx} = \frac{3x^2y^2-2x}{3y^2-2x^3y}$$

slope at $(x,y)=(2,-1)$ $= \left.\frac{dy}{dx}\right|_{(x,y)=(2,-1)} = \frac{3(2)^2(-1)^2-2(2)}{3(-1)^2-2(2)^3(-1)} = \frac{8}{19}$

point: $(2,-1)$

tangent line: $y-(-1) = \frac{8}{19}(x-2)$

$$y = \frac{8}{19}(x-2)-1$$

$$y = \frac{8}{19}x - \frac{35}{19}$$

4. (2 points) Compute $\frac{dy}{dx}$ for the given function. Write your answer completely in terms of x.

$$y = \left(1/x^2\right)^{1/x^3}$$

$y = (x^{-2})^{1/x^3} = x^{-2/x^3}$ (simplification is not required, but it makes the problem a little easier.)

$\ln(y) = \ln(x^{-2/x^3})$

$\ln(y) = -\frac{2}{x^3} \cdot \ln(x) = \frac{-2\ln(x)}{x^3}$

$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(\frac{-2\ln(x)}{x^3}\right)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{\frac{d}{dx}(-2\ln(x)) \cdot x^3 - (-2\ln(x)) \cdot \frac{d}{dx}(x^3)}{(x^3)^2}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{-2 \cdot \frac{1}{x} \cdot x^3 + 2\ln(x) \cdot 3x^2}{x^6}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{-2x^2 + 6x^2\ln(x)}{x^6}$

$\frac{dy}{dx} = y \cdot \frac{-2x^2 + 6x^2\ln(x)}{x^6}$

$\frac{dy}{dx} = x^{-2/x^3} \cdot \frac{-2x^2 + 6x^2\ln(x)}{x^6}$

$\frac{dy}{dx} = x^{-2/x^3} \cdot \frac{-2 + 6\ln(x)}{x^4}$