Math 220 (section AD?) **Quiz 4** **Spring 2017**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) What is the slope of the curve $y = 2x^4e^x$ at the point $\left(-1, \frac{2}{e}\right)$?

$\frac{dy}{dx} = \frac{d}{dx}(2x^4) \cdot e^x + 2x^4 \cdot \frac{d}{dx}(e^x)$ (product rule)

$\frac{dy}{dx} = 8x^3 \cdot e^x + 2x^4 \cdot e^x$

slope at $x=-1$ is $\left.\frac{dy}{dx}\right|_{x=-1} = 8(-1)^3e^{-1} + 2(-1)^4e^{-1} = -6e^{-1} = \frac{-6}{e}$

2. (2 points) Determine the equation of the line which is tangent to the curve $f(x) = 4 + \sqrt{x}$ and parallel to the line $x - 6y = 60$.

$x - 6y = 60 \Rightarrow 6y = x - 60 \Rightarrow y = \frac{1}{6}x - 10$

This line has slope $\frac{1}{6}$, since the tangent line is parallel, it also has slope $\frac{1}{6}$

$f(x) = 4 + \sqrt{x} = 4 + x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

setting $f'(x) = \frac{1}{6}$ gives $\frac{1}{2\sqrt{x}} = \frac{1}{6} \Rightarrow x = 9$

Point: $(9, f(9)) = (9, 4+\sqrt{9}) = (9, 7)$

slope: $1/6$

tangent line: $y - 7 = \frac{1}{6}(x-9)$

$y = \frac{1}{6}(x-9) + 7$

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $p = \left(\frac{x^2}{\sqrt[6]{x^3}} - \frac{x^4}{x\sqrt{x}}\right)^2 + 5\ln(3)e^{\pi/2}$ (simplify your answer)

$$p = \left(\frac{x^2}{x^{3/6}} - \frac{x^4}{x^{3/2}}\right)^2 + \text{constant}$$

$$p = (x^{3/2} - x^{5/2})^2 + \text{constant}$$

$$p = (x^{3/2})^2 - 2x^{3/2}x^{5/2} + (x^{5/2})^2 + \text{constant}$$

$$p = x^3 - 2x^4 + x^5 + \text{constant}$$

$$\frac{dp}{dx} = 3x^2 - 8x^3 + 5x^4$$

(b) $Z = \csc(\theta)\left(10\theta^6 + \frac{1}{\theta}\right)$ $= \csc(\theta)(10\theta^6 + \theta^{-1})$

$$\frac{dZ}{d\theta} = \frac{d}{d\theta}(\csc(\theta)) \cdot (10\theta^6 + \theta^{-1}) + \csc(\theta) \cdot \frac{d}{d\theta}(10\theta^6 + \theta^{-1})$$

$$\frac{dZ}{d\theta} = -\csc(\theta)\cot(\theta)(10\theta^6 + \theta^{-1}) + \csc(\theta)(60\theta^5 - \theta^{-2})$$

(c) $L = \frac{8\tan(t) + 6t}{t^2 + 9}$

$$\frac{dL}{dt} = \frac{\frac{d}{dt}(8\tan(t) + 6t) \cdot (t^2 + 9) - (8\tan(t) + 6t) \cdot \frac{d}{dt}(t^2 + 9)}{(t^2 + 9)^2}$$

$$\frac{dL}{dt} = \frac{(8\sec^2(t) + 6)(t^2 + 9) - (8\tan(t) + 6t)(2t)}{(t^2 + 9)^2}$$