Math 220 (section AD?) Quiz 3 Spring 2017

Name Solutions

- 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Determine an equation for each horizontal asymptote on the graph of the following function. Your answer must be justified using limits.

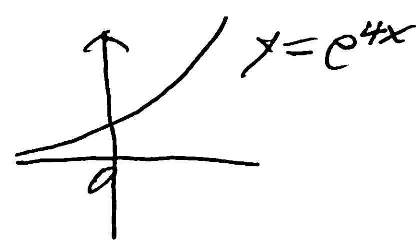
$$f(x) = \frac{18 - 35e^{4x}}{7e^{4x} + 6}$$

$$\lim_{x\to\infty} \frac{\overbrace{18-35e^{4x}}^{\to -\infty}}{\underbrace{7e^{4x}+6}_{\to \infty}} = \lim_{x\to\infty} \frac{e^{4x}\left(\frac{18}{e^{4x}} - 35\right)}{e^{4x}\left(7 + 6/e^{4x}\right)}$$

$$= \lim_{x\to\infty} \frac{18/e^{4x} - 35}{7 + 6/e^{4x}}$$

$$= \frac{-35}{7} = -5$$

$$\lim_{x\to-\infty} \frac{18-35e^{4x}}{7e^{4x}+6} = \frac{18}{6} = 3 \quad \text{since } \lim_{x\to-\infty} e^{4x} = 0$$



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horizontal asymptotes are

$y=-5$ and $y=3$

2. (2 points) Evaluate $\tan(2 \arcsin(3/5))$.

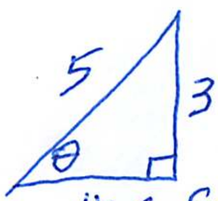
Let $\theta = \arcsin(3/5)$

Then $\sin(\theta) = 3/5$ $\left(\frac{\text{opp}}{\text{hyp}}\right)$

range of $\arcsin(\theta)$ is $[-\pi/2, \pi/2]$

but $0 < \frac{3}{5} < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{2}$



4 ← from Pythagorean Theorem

$$\tan(2\arcsin(3/5)) = \tan(2\theta)$$

$$= \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$= \frac{2\sin(\theta)\cos(\theta)}{\cos^2(\theta) - \sin^2(\theta)}$$

$$= \frac{2(3/5)(4/5)}{(4/5)^2 - (3/5)^2}$$

$$= 24/7$$

3. (2 points each) Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is ∞ or $-\infty$.

(a) $\displaystyle\lim_{x \to 0^+} \frac{\cos(4x^2 + \pi)}{1 - e^x}$ $\begin{matrix} \to -1 \\ \to 0^- \end{matrix}$ $= \infty$

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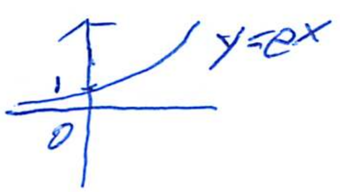
reasoning

$\lim_{x \to 0^+} \cos(4x^2 + \pi) = \cos(4 \cdot 0^2 + \pi) = \cos(\pi) = -1$

$\lim_{x \to 0^+} (1 - e^x) = 1 - e^0 = 1 - 1 = 0$ (is it 0^+ or 0^- ?)



$x \to 0^+$ so for

$x > 0$, note that $e^x > 1$

thus $1 - e^x < 0$

and $\lim_{x \to 0^+} (1 - e^x) = 0^-$

(b) $\lim_{x\to 5} \frac{5-x}{3-\sqrt{x+4}}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$ $= \lim_{x\to 5} \frac{5-x}{3-\sqrt{x+4}} \cdot \frac{3+\sqrt{x+4}}{3+\sqrt{x+4}}$

$= \lim_{x\to 5} \frac{(5-x)(3+\sqrt{x+4})}{3^2-(\sqrt{x+4})^2}$

$= \lim_{x\to 5} \frac{(5-x)(3+\sqrt{x+4})}{9-(x+4)}$

$= \lim_{x\to 5} \frac{(5-x)(3+\sqrt{x+4})}{5-x}$

$= \lim_{x\to 5} (3+\sqrt{x+4})$

$= 3+\sqrt{5+4}$

$= 6$

4. (1 point) Fill in the box to complete the definition of the term *continuous*.

A function w is *continuous* at a number b if and only if

$$\boxed{\lim_{x\to b} w(x) = w(b)}$$