Math 220 **Quiz 12 (take-home)** **Spring 2017**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Andrew McConvey
- ▷ **AD2**, TR 1:00-2:50, Sarah Loeb
- ▷ **ADA**, TR 8:00-8:50, Christopher Linden
- ▷ **ADB**, TR 9:00-9:50, Dakota Ihli
- ▷ **ADC**, TR 10:00-10:50, Cassie Christenson
- ▷ **ADD**, TR 11:00-11:50, Daulet Dyussekenov
- ▷ **ADE**, TR 12:00-12:50, Daulet Dyussekenov
- ▷ **ADF**, TR 1:00-1:50, Cassie Christenson
- ▷ **ADG**, TR 2:00-2:50, Xinghua Gao
- ▷ **ADH**, TR 3:00-3:50, Xinghua Gao
- ▷ **ADJ**, TR 9:00-9:50, Lan Wang
- ▷ **ADK**, TR 10:00-10:50, Lan Wang
- ▷ **ADO**, TR 2:00-2:50, Christopher Linden
- ▷ **ADQ**, TR 4:00-4:50, Dakota Ihli

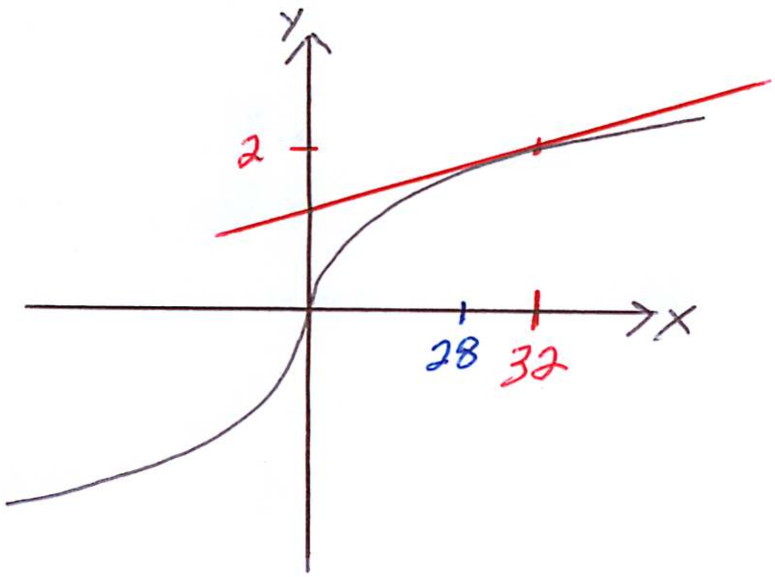
- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- You may use a calculator only for basic arithmetic. In particular you should not use its graphing features.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official lecture period on Friday, April 21.**
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until the quizzes have been collected for all MATH 220 lectures (11am, 1pm, 3pm).**

1. (3 points) A calculator gives an estimate of 1.947294361 for the value of $\sqrt[5]{28}$.

Using the techniques of linear approximation found in section 3.10, show that you are able to obtain a very similar estimate of 1.95 without the use of any technology.

$f(x) = \sqrt[5]{x} = x^{1/5}$

$f'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$



$L(x) = 2 + \frac{1}{80}(x - 32)$

tangent line at $x = 32$

point: $(32, f(32)) = (32, \sqrt[5]{32}) = (32, 2)$

slope: $f'(32) = \frac{1}{5 \cdot 32^{4/5}} = \frac{1}{80}$

tangent line: $y - 2 = \frac{1}{80}(x - 32)$

$y = 2 + \frac{1}{80}(x - 32)$

$f(x) \approx L(x)$ near point of tangency

$\sqrt[5]{x} \approx 2 + \frac{1}{80}(x - 32)$ for x near 32

$\sqrt[5]{28} \approx 2 + \frac{1}{80}(28 - 32)$

$\sqrt[5]{28} \approx 1.95$ (seen as overestimate from graphs)

2. (3 points) Suppose that the polynomial $h(t)$ is an even function and satisfies the following conditions.

- $h(5) = 40$
- $h'(5) = 10$
- $h''(5) = 8$
- $h'''(5) = 3$

Use properties of even functions along with the techniques of linear approximation found in section 3.10 to estimate the value of $h(-5.2)$. Simplify your answer.

Since h is even, $h(-5.2) = h(5.2)$

Tangent line at $t=5$

Point: $(5, h(5)) = (5, 40)$

slope: $h'(5) = 10$

tangent line: $y - 40 = 10(t-5)$

$y = 40 + 10(t-5)$

$h(t) \approx 40 + 10(t-5)$ for t near 5.

$h(5.2) \approx 40 + 10(5.2 - 5)$

$h(5.2) \approx 42$

Thus $h(-5.2) = h(5.2) \approx 42$

3. (4 points) One of the three points of intersection for the graphs of $y = \frac{50}{x}$ and $y = x^2 - 200$ occurs at a positive x-value. Approximate this x-value by using Newton's Method with an initial estimate of $x_1 = 20$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$$x^2 - 200 = \frac{50}{x} \Rightarrow x^3 - 200x = 50 \Rightarrow x^3 - 200x - 50 = 0$$

Let $f(x) = x^3 - 200x - 50$ and apply Newton's Method

$$f(x) = x^3 - 200x - 50 \Rightarrow f'(x) = 3x^2 - 200$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 200x_n - 50}{3x_n^2 - 200}$$

$x_1 = 20$ given as first estimate

$$x_2 = x_1 - \frac{x_1^3 - 200x_1 - 50}{3x_1^2 - 200} = 20 - \frac{20^3 - 200(20) - 50}{3(20)^2 - 200}$$

$x_2 = 16.05$

$$x_3 = x_2 - \frac{x_2^3 - 200x_2 - 50}{3x_2^2 - 200}$$

$$x_3 \approx 16.05 - \frac{(16.05)^3 - 200(16.05) - 50}{3(16.05)^2 - 200}$$

$x_3 \approx 14.52327396$

$$x_4 = x_3 - \frac{x_3^3 - 200x_3 - 50}{3x_3^2 - 200}$$

$$x_4 \approx 14.52327396 - \frac{(14.52327396)^3 - 200(14.52327396) - 50}{3(14.52327396)^2 - 200}$$

$x_4 \approx 14.27216549$