Math 220 (section AD?) Quiz 11 Spring 2017

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = \dfrac{x^2}{\sqrt{2x^3+9}}$ on the interval $[0, 2]$. Simplify your answer.

$$f_{ave} = \frac{1}{2-0}\int_0^2 \frac{x^2}{\sqrt{2x^3+9}}\,dx$$

$$= \frac{1}{2}\int_0^2 \frac{1}{\sqrt{2x^3+9}}\cdot x^2\,dx$$

$$u = 2x^3+9$$
$$du = 6x^2\,dx$$
$$\frac{1}{6}du = x^2\,dx$$
$$x=0 \Rightarrow u = 2\cdot 0^3+9 = 9$$
$$x=2 \Rightarrow u = 2\cdot 2^3+9 = 25$$

$$= \frac{1}{2}\int_9^{25} \frac{1}{\sqrt{u}}\cdot\frac{1}{6}\,du$$

$$= \frac{1}{12}\int_9^{25} u^{-1/2}\,du$$

$$= \frac{1}{12}\left[\frac{1}{1/2}u^{1/2}\right]_9^{25}$$

$$= \frac{1}{12}\left[2\sqrt{u}\right]_9^{25}$$

$$= \frac{1}{12}\left[2\sqrt{25} - 2\sqrt{9}\right]$$

$$= \frac{1}{12}\left[10 - 6\right]$$

$$= \frac{4}{12}$$

$$= 1/3$$

2. Let **R** be the finite region bounded by the graphs of the following functions.

$$y = -2x + 14$$

$$y = \frac{20}{x}$$

intersection

$-2x+14 = \frac{20}{x}$

$-2x^2+14x = 20$

$-2x^2+14x-20=0$

$-2(x^2-7x+10)=0$

$-2(x-2)(x-5)=0$

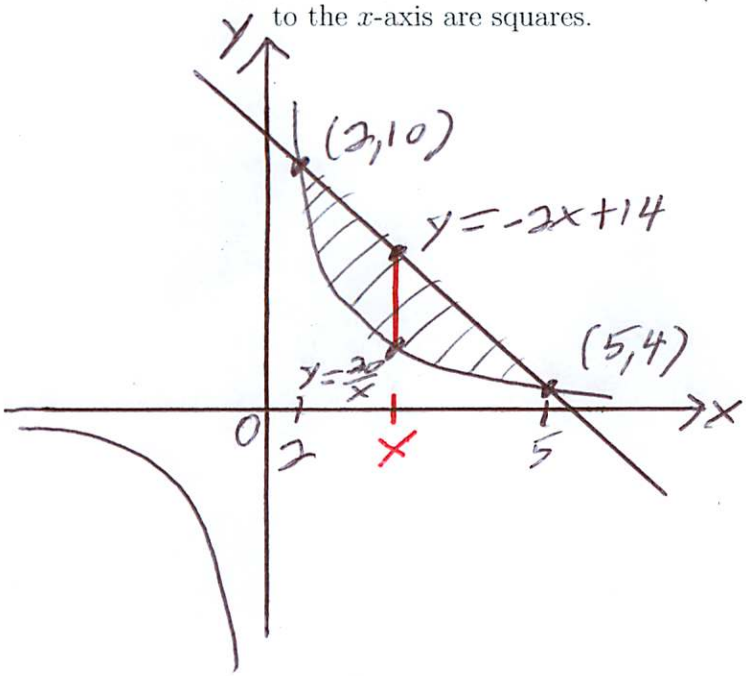
$x=2$ or $x=5$

Points $(2,10)$ and $(5,4)$

Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

(a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are squares.

not drawn to scale



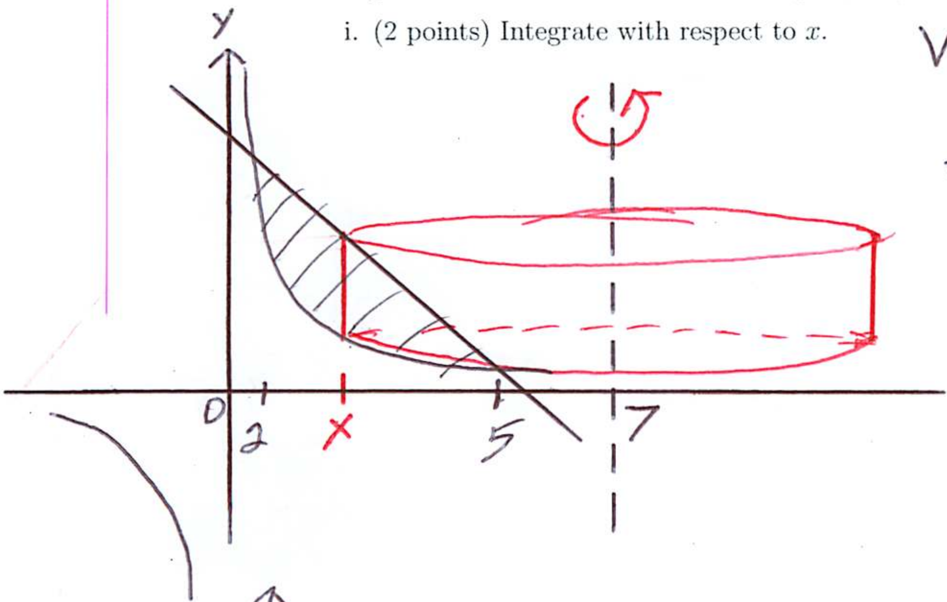
$$V = \int_2^5 \left(\begin{matrix}\text{cross-sectional}\\ \text{area}\end{matrix}\right) dx$$

$$= \int_2^5 (\text{side})^2\, dx$$

$$= \int_2^5 \left(-2x+14-\frac{20}{x}\right)^2 dx$$

(b) The volume of the solid formed when **R** is revolved around the line $x = 7$. Set up the integrals for this volume in the following two ways.

i. (2 points) Integrate with respect to x.



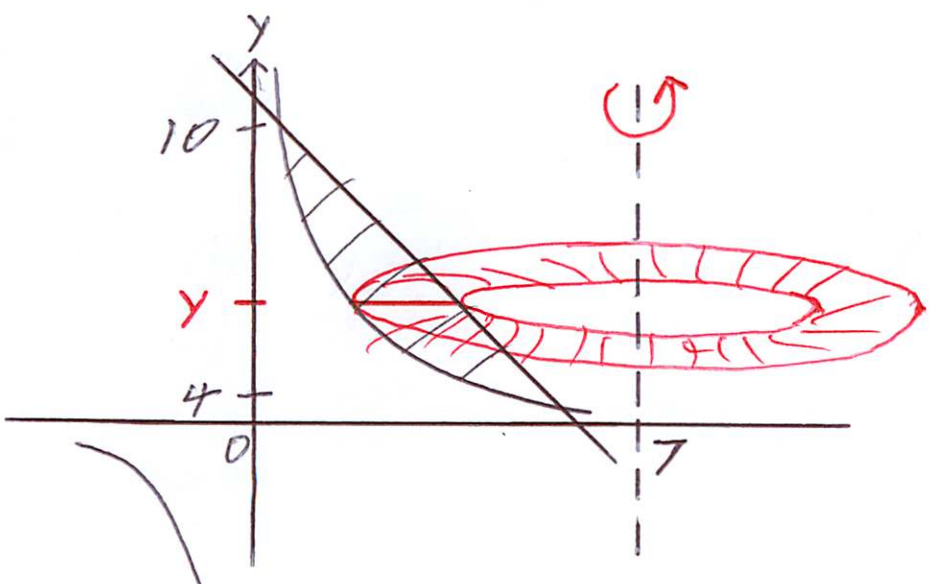
$$V = \int_2^5 \left(\text{surface area}\right) dx$$

$$= \int_2^5 2\pi r h \, dx$$

$$= \int_2^5 2\pi(7-x)\left(-2x+14-\frac{20}{x}\right) dx$$

not drawn to scale

ii. (2 points) Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)



$$V = \int_4^{10} \left(\text{cross-sectional area}\right) dy$$

$$= \int_4^{10} \left(\pi r_{out}^2 - \pi r_{in}^2\right) dy$$

$$= \int_4^{10} \left(\pi\left(7-\frac{20}{y}\right)^2 - \pi\left(7-\frac{14-y}{2}\right)^2\right) dy$$

note

$y = \frac{20}{x} \Rightarrow x = \frac{20}{y}$

$y = -2x+14 \Rightarrow x = \frac{14-y}{2}$

cross-section at y-slice