Math 220 (section AD?) **Quiz 10** **Spring 2017**

Name: solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Precisely state *The Mean Value Theorem.*

Suppose f is continuous on $[a,b]$ and differentiable on (a,b). Then there is a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_0^{\sqrt{3}} 18t\sqrt{t^2+1}\,dt = \int_0^{\sqrt{3}} 9\sqrt{t^2+1}\cdot 2t\,dt$$

$$\begin{array}{l} u = t^2+1 \\ du = 2t\,dt \\ \hline t=0 \Rightarrow u = 0^2+1 = 1 \\ t=\sqrt{3} \Rightarrow u = (\sqrt{3})^2+1 = 4 \end{array}$$

$$= \int_1^4 9\sqrt{u}\,du$$

$$= \int_1^4 9u^{1/2}\,du$$

$$= \left[9\cdot\frac{1}{3/2}u^{3/2}\right]_1^4$$

$$= \left[6u^{3/2}\right]_1^4$$

$$= 6(4)^{3/2} - 6(1)^{3/2}$$

$$= 6\cdot 8 - 6$$

$$= 42$$

3. (2 points) Evaluate the indefinite integral.

$$\int \frac{60e^{3x}}{(e^{3x}+2)^{11}}\,dx = \int \frac{20}{(e^{3x}+2)^{11}} \cdot 3e^{3x}\,dx$$

$$\left(\begin{array}{l} u = e^{3x}+2 \\ du = 3e^{3x}\,dx \end{array} \right)$$

$$= \int \frac{20}{u^{11}}\,du$$

$$= \int 20u^{-11}\,du$$

$$= 20 \cdot \frac{1}{-10} u^{-10} + C$$

$$= -2u^{-10} + C$$

$$= -2(e^{3x}+2)^{-10} + C$$

$$= \frac{-2}{(e^{3x}+2)^{10}} + C$$

4. (2 points) Evaluate the indefinite integral.

$$\int 4\sin^7(x)\cos(x)\left(\sin^4(x)+5\right)^{100}dx = \int 4\sin^7(x)\left(\sin^4(x)+5\right)^{100}\cdot\cos(x)\,dx$$

$$\begin{pmatrix} u=\sin(x) \\ du=\cos(x)dx \end{pmatrix}$$

$$=\int 4u^7(u^4+5)^{100}du$$

$$=\int u^4(u^4+5)^{100}\cdot 4u^3du$$

$$\begin{pmatrix} w=u^4+5 \left(\text{so } u^4=w-5\right) \\ dw=4u^3du \end{pmatrix}$$

$$=\int (w-5)\,w^{100}dw$$

$$=\int \left(w^{101}-5w^{100}\right)dw$$

$$=\frac{1}{102}w^{102}-\frac{5}{101}w^{101}+C$$

$$=\frac{1}{102}(u^4+5)^{102}-\frac{5}{101}(u^4+5)^{101}+C$$

$$=\frac{1}{102}\left(\sin^4(x)+5\right)^{102}-\frac{5}{101}\left(\sin^4(x)+5\right)^{101}+C$$

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or you could make one substitution

$u=\sin^4(x)+5$

5. (2 points) Let **R** be the finite region bounded by the given functions. In the following way, set up <u>but do not evaluate</u> definite integrals which represent the area of the region **R**.

$y = 3e^{2x}$

$y = -4x + 3$

$y = 27$

intersections

(1) $3e^{2x}$ and $-4x+3$ intersect at y-intercept $(0,3)$

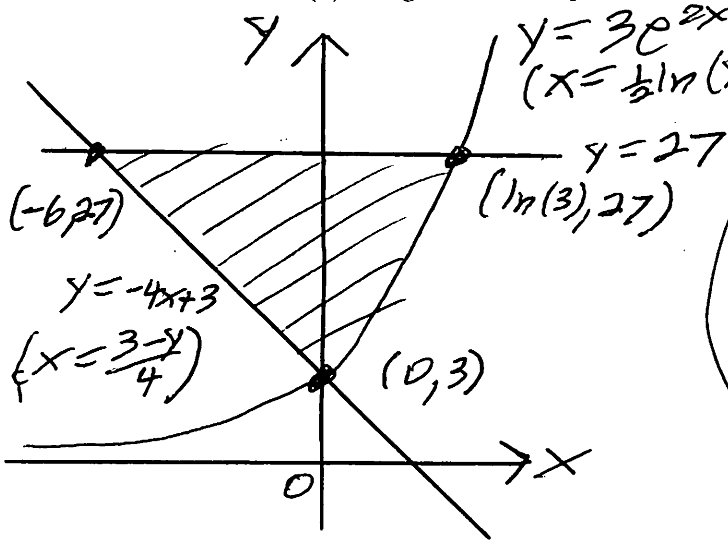
(2) $-4x+3=27 \Rightarrow x=-6$ (point $(-6,27)$)

(3) $3e^{2x}=27 \Rightarrow e^{2x}=9 \Rightarrow 2x=\ln(9)=\ln(3^2)=2\ln(3)$

so $x=\ln(3)$

point $(\ln(3),27)$

(a) Integrate with respect to x.



$$\text{area} = \int_{-6}^{0}\left(27-(-4x+3)\right)dx + \int_{0}^{\ln(3)}\left(27-3e^{2x}\right)dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$y=3e^{2x} \Rightarrow e^{2x}=\frac{y}{3} \Rightarrow \ln(e^{2x})=\ln(y/3)$

$\Rightarrow 2x = \ln(y/3)$

so $x = \frac{1}{2}\ln(y/3)$

$y=-4x+3 \Rightarrow 4x=3-y \Rightarrow x=\frac{3-y}{4}$

$$\text{area} = \int_{3}^{27}\left(\frac{1}{2}\ln(y/3) - \frac{3-y}{4}\right)dy$$