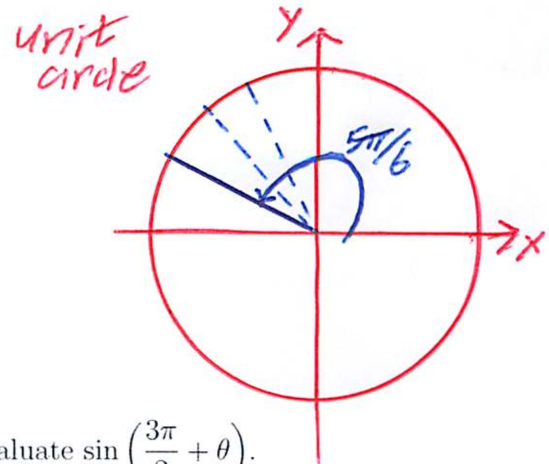
Math 220 (section AD?) Quiz 1 Spring 2017

Name Solutions

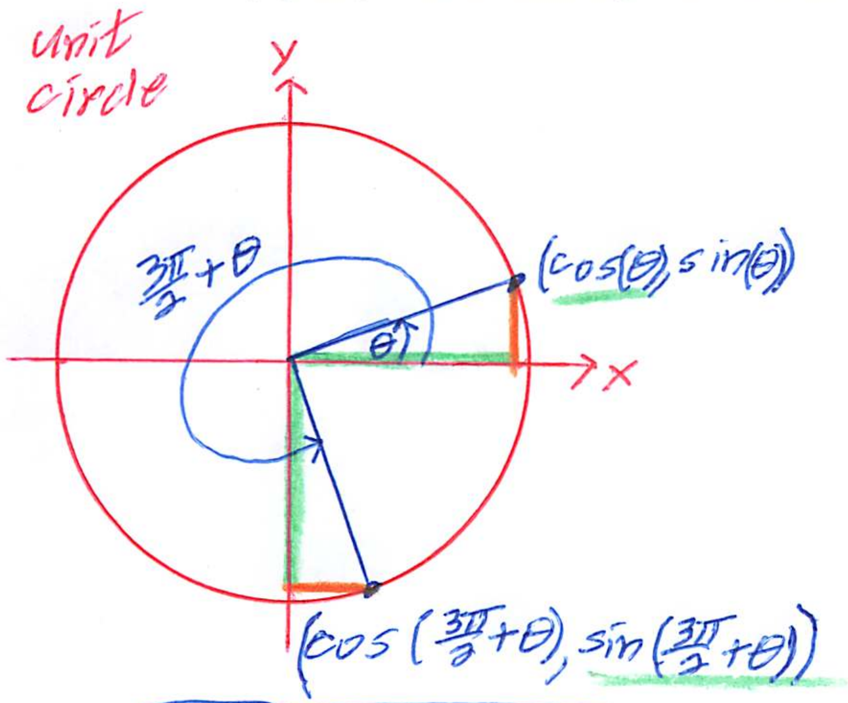
- You have 20 minutes
- No calculators
- Show sufficient work

1. (1 point) Evaluate $\cot\left(\frac{5\pi}{6}\right)$.



$$\cot(5\pi/6) = \frac{\cos(5\pi/6)}{\sin(5\pi/6)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

2. (3 points) Given an acute angle θ for which $\sin(\theta) = 1/4$, evaluate $\sin\left(\frac{3\pi}{2} + \theta\right)$.



$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\left(\frac{1}{4}\right)^2 + \cos^2(\theta) = 1$$
$$\cos^2(\theta) = \frac{15}{16}$$
$$\cos(\theta) = \pm\sqrt{\frac{15}{16}}$$

θ acute $\Rightarrow$

$$\cos(\theta) = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$$

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos(\theta) = -\frac{\sqrt{15}}{4}$$

3. (3 points) Determine the domain of the given function.

$$f(x) = \frac{\sqrt{3x-15}}{6-\sqrt{100-x^2}}$$

From $\sqrt{3x-15}$ we get $3x-15 \geq 0 \Rightarrow 3x \geq 15 \Rightarrow x \geq 5$.

From $\sqrt{100-x^2}$ we get $100-x^2 \geq 0 \Rightarrow 100 \geq x^2 \Rightarrow$
$x^2 \leq 100 \Rightarrow \sqrt{x^2} \leq \sqrt{100} \Rightarrow |x| \leq 10 \Rightarrow -10 \leq x \leq 10$

The denominator equals 0 when
$6-\sqrt{100-x^2} = 0 \Rightarrow 6 = \sqrt{100-x^2} \Rightarrow 36 = 100-x^2$
$\Rightarrow x^2 = 64 \Rightarrow x = \pm 8$

Thus $x \neq \pm 8$

$x \geq 5$ and $-10 \leq x \leq 10$ and $x \neq \pm 8$ so domain of $f(x)$ is $[5,8) \cup (8,10]$

4. (3 points) Suppose that $f(x)$ and $g(x)$ are both odd functions and let $h(x) = f(x) \cdot \cos(g(x))$. Carefully prove that $h(x)$ is an odd function.

$h(-x) = f(-x) \cdot \cos(g(-x))$

$= -f(x) \cdot \cos(-g(x))$ ($f(-x) = -f(x)$ since f is odd) ($g(-x) = -g(x)$ since g is odd)

$= -f(x) \cdot \cos(g(x))$ since cosine is even

$= -h(x)$

Thus $h(x)$ is an odd function