• Sit in your assigned seat (circled below).
• Circle your TA discussion section.
• Do not open this test booklet until I say START.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• Remove hats and sunglasses.
• You must show sufficient work to justify each answer.
• While the test is in progress, we will not answer questions concerning the test material.
• Do not leave early unless you are at the end of a row.
• Quit working and close this test booklet when I say STOP.
• Quickly turn in your test to me or a TA and show your Student ID.

> AD1, TR 9:00-10:50, Hannah Burson  > ADH, TR 3:00-3:50, Dara Zirlin
> AD2, TR 1:00-2:50, Cassie Christenson  > ADJ, TR 9:00-9:50, Xujun 'Henry' Liu
> ADA, TR 8:00-8:50, Iftikhar Ahmed  > ADK, TR 10:00-10:50, Xujun 'Henry' Liu
> ADB, TR 9:00-9:50, Iftikhar Ahmed  > ADL, TR 11:00-11:50, Jooyeon 'Jane' Chung
> ADC, TR 10:00-10:50, Elizabeth 'Liz' Tatum  > ADM, TR 12:00-12:50, Jooyeon 'Jane' Chung
> ADD, TR 11:00-11:50, Elizabeth 'Liz' Tatum  > ADN, TR 1:00-1:50, Xiaolong 'Hans' Han
> ADE, TR 12:00-12:50, Emily Heath  > ADO, TR 2:00-2:50, Martino Fassina
> ADF, TR 1:00-1:50, Emily Heath  > ADP, TR 3:00-3:50, Martino Fassina
> ADG, TR 2:00-2:50, Dara Zirlin  > ADQ, TR 4:00-4:50, Xiaolong 'Hans' Han

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
1 2 3 4 5 6 K
1 2 3 4 5 6 J J
1 3 2 4 5 6 I I
1 2 3 4 5 6 H H
1 2 3 4 5 6 G G
1 2 3 4 5 6 F F
1 2 3 4 5 6 E E
1 2 3 4 5 6 D D
1 2 3 4 5 6 C C
1 2 3 4 5 6 B B
A 1 2 3

FRONT OF ROOM – 100 Materials Science and Engineering Building
1. (10 points) Use a linear approximation to estimate \( \ln(7/8) \). Write your answer as either a simplified fraction or a decimal value.

\[ L(x) = x - 1 \]

we will find the line \( L(x) \)

which is tangent to \( f(x) = \ln(x) \)

at \( x = 1 \).

Point: \( (1, f(1)) = (1, \ln(1)) = (1, 0) \)

slope: \( f'(1) = \frac{1}{1} = 1 \) (since \( f'(x) = \frac{1}{x} \))

tangent line: \( y - 0 = 1 \cdot (x - 1) \)

\[ L(x) = x - 1 \]

\( \ln(7/8) \approx x - 1 \) for \( x \) near 1

\( \ln(\frac{7}{8}) \approx \frac{2}{8} - 1 \)

\( \ln(\frac{7}{8}) \approx -\frac{1}{8} = -0.125 \)

2. (10 points) Suppose that \( f(x) \) is a polynomial which satisfies the following conditions.

- \( \int_{7}^{15} f(x) \, dx = 23 \)
- \( \int_{21}^{15} f(x) \, dx = 13 \)

Evaluate the following quantities.

(a) \( \int_{7}^{21} (20f(x) + 3) \, dx \)

\[ = 20 \int_{7}^{21} f(x) \, dx + \int_{7}^{21} 3 \, dx \]

\[ = 20 \cdot 10 + [3x]_{7}^{21} = 200 + 3\cdot 21 - 3\cdot 7 = 242 \]

(b) \( \int_{1}^{2} 54x^2 f(2x^3 + 5) \, dx \)

\[ = \int_{1}^{2} 9f(u) \, du = 9 \int_{7}^{21} f(u) \, du \]

\[ = 9 \cdot 10 = 90 \]

\( u = 2x^3 + 5 \)

\( du = 6x^2 \, dx \)

\[ 9 \int_{7}^{21} f(u) \, du \]

\( x = 1 \Rightarrow u = 7 \)

\( x = 2 \Rightarrow u = 21 \)
3. (10 points) Let \( g(x) = \int_{\sin(9x)}^{10} \frac{1}{t^{16} + 1} \, dt \). Find \( g'(x) \).

\[
g(x) = -\int_{\sin(9x)}^{10} \frac{1}{t^{16} + 1} \, dt
\]

\[
g'(x) = -\frac{1}{\sin^{16}(9x) + 1} \cdot \cos(9x) \cdot 9
\]

more formally,
let \( u = \sin(9x) \) so \( g = -\int_{\sin(9x)}^{10} \frac{1}{t^{16} + 1} \, dt \)

\[
\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = -\frac{1}{u^{16} + 1} \cdot \cos(9x) \cdot 9 = \frac{1}{\sin^{16}(9x) + 1} \cdot \cos(9x) \cdot 9
\]

4. (10 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit. Simplify your answer.

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{60k + 12n}{n^2} \right) = \lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{60k + 12n}{n^2} \right)
\]

\[
= \lim_{n \to \infty} \left( \sum_{k=1}^{n} \frac{60k + 12n}{n^2} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{60 \sum_{k=1}^{n} k + 12n \cdot n}{n^2} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{60 \left( \frac{n(n+1)}{2} \right) + 12n}{n^2} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{30(n^2 + n)}{n^2} + 12 \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{30n^2 + 30n}{n^2} + 12 \right)
\]

\[
= 30 + 30 + 12 = 42
\]
5. (10 points) Let \( R \) be the finite region bounded by the graphs of \( y = \frac{1}{5}x \) and \( y = \sqrt{x} \). These curves intersect at the origin and at the point \((x, y) = (25, 5)\). Revolve \( R \) around the horizontal line \( y = 8 \) to form a solid. In the following manner, set up but do not evaluate definite integrals which represent the volume of the solid. Use proper notation.

(a) Integrate with respect to \( x \).

\[
V = \int_0^{25} \left( \text{area cross-sectional} \right) \, dx
= \int_0^{25} \left( \pi \text{out}^2 - \pi \text{in}^2 \right) \, dx
= \int_0^{25} \left( \pi (8 - \frac{1}{5}x)^2 - \pi (8 - \sqrt{x})^2 \right) \, dx
\]

(b) Integrate with respect to \( y \). (The integrands in parts (a) and (b) should be different.)

\[
V = \int_0^5 \left( \text{area surface} \right) \, dy
= \int_0^5 2\pi \cdot \text{rad. height} \, dy
= \int_0^5 2\pi (8-y)(5y-y^2) \, dy
\]

Note: \( y = \frac{1}{5}x \) \( \Rightarrow \) \( x = 5y \)
\[ y = \sqrt{x} \] \( \Rightarrow \) \( x = y^2 \)
6. (10 points) Determine the formula for a function \( f(x) \) such that \( f''(x) = 630e^{3x} + 40 \cos x \), \( f'(0) = 20 \) and \( f(0) = 80 \).

\[
f'(x) = \int f''(x) \, dx = \int (630e^{3x} + 40\cos x) \, dx = 210e^{3x} + 40\sin x + C
\]

\( f'(0) = 20 \Rightarrow 210e^{0} + 40\sin(0) + C = 20 \)

\( \Rightarrow C = -190 \)

\( \Rightarrow f'(x) = 210e^{3x} + 40\sin x - 190 \)

\[
f(x) = \int f'(x) \, dx = \int (210e^{3x} + 40\sin x - 190) \, dx = 70e^{3x} - 40\cos x - 190x + D
\]

\( f(0) = 80 \Rightarrow 70e^{0} - 40\cos(0) - 190(0) + D = 80 \)

\( \Rightarrow D = 50 \)

\( \Rightarrow f(x) = 70e^{3x} - 40\cos x - 190x + 50 \)

7. (10 points) Find the average value of the function \( f(x) = \frac{32x}{\sqrt{2x^2 + 49}} \) on the interval \([0, 4]\). Simplify your answer.

\[
f_{ave} = \frac{1}{4 - 0} \int_{0}^{4} \frac{32x}{\sqrt{2x^2 + 49}} \, dx
\]

\[
= \frac{1}{4} \int_{0}^{4} \frac{8}{\sqrt{u}} \, du
\]

\[
= 2 \left[ \frac{1}{12} u^{\frac{1}{2}} \right]_{49}^{81}
\]

\[
= 2 \left[ 2\sqrt{81} - 2\sqrt{49} \right]
\]

\[
= 2 \cdot 2 \cdot 9 - 2 \cdot 7
\]

\[
= 8
\]
8. (10 points) Evaluate the indefinite integral.

\[ \int \frac{9x^2 + 84x + 9}{x^2 + 1} \, dx \]

\[ = \int \left( 9 + \frac{84x}{x^2 + 1} \right) \, dx \]

\[ = 9 \int dx + \int \frac{84x}{x^2 + 1} \, dx \]

\[ = 9x + \int \frac{42}{u} \, du \quad \text{with} \quad u = x^2 + 1, \quad du = 2x \, dx \]

\[ = 9x + 42 \ln |u| + C \]

\[ = 9x + 42 \ln (x^2 + 1) + C \]

9. (10 points) Evaluate the indefinite integral.

\[ \int \sec^3(x) \tan^3(x) \, dx \]

\[ = \int \sec^3(x) \tan^2(x) \sec(x) \, dx \]

\[ = \int \sec^3(x)(\sec^2(x)-1) \sec(x) \, dx \]

\[ = \int \sec^6(x) - \sec^4(x) \, dx \]

\[ = \int \left( u^6 - u^4 \right) \, du \quad \text{with} \quad u = \sec(x), \quad du = \sec(x) \tan(x) \, dx \]

\[ = \frac{1}{39} u^9 - \frac{1}{37} u^7 + C \]

\[ = \frac{1}{39} \sec^9(x) - \frac{1}{37} \sec^7(x) + C \]

Could also write integrand in terms of \( \sin(x) \) and \( \cos(x) \), then use \( u = \cos(x) \).
10. (10 points) Evaluate the indefinite integral.

\[ \int \frac{91x^{12}}{x^{26} + 1} \, dx = \int \frac{91}{(x^{13})^2 + 1} \, dx \]

Let \( u = x^{13} \)

\[ du = 13x^{12} \, dx \]

\[ 7du = 91x^{12} \, dx \]

\[ = \int \frac{7}{u^2 + 1} \, du \]

\[ = 7 \arctan(u) + C \]

\[ = 7 \arctan(x^{13}) + C \]
Students - do not write on this page!

1. (10 points) ____________________
2. (10 points) ____________________
3. (10 points) ____________________
4. (10 points) ____________________
5. (10 points) ____________________
6. (10 points) ____________________
7. (10 points) ____________________
8. (10 points) ____________________
9. (10 points) ____________________
10. (10 points) ____________________

**TOTAL (100 points) ______________**